Section 8: The Term Structure of Interest Rates

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1. Basic structure of the notes

- High-level summary of theoretical frameworks to interpret empirical facts.
- Per asset class, we will discuss:
  1. Key empirical facts in terms of prices (unconditional and conditional risk premia) and asset ownership.
  2. Interpret the facts using the theoretical frameworks.
  3. Facts and theories linking financial markets and the real economy.
  4. Active areas of research and some potentially interesting directions for future research.
- The notes cover the following asset classes:
  1. Equities (weeks 1-5).
     - Discount rates and the term structure of risk (week 1)
     - The Cross-section and the factor zoo (week 2)
     - Intermediary-based Asset Pricing (week 3)
     - Production-based asset pricing (week 4)
     - Demand-based asset pricing (week 5)
  2. Mutual funds and hedge funds (week 6).
  3. Volatility (week 7).
  4. Government bonds (week 8).
  5. Corporate bonds (week 9).
  6. Currencies (week 10).
  7. Commodities (week 11).
  8. Real estate (week 12).

2. Government Bonds

2.1. Notation

- $P_t(n)$ is the price of an $n-$year nominal bond at time t with payoff 1.

- $y_t(n)$ is the corresponding (continuously-compounded) bond yield, or **yield-to-maturity**

$$y_t(n) = -\frac{1}{n}\log P_t(n) = -\frac{1}{n}p_t(n).$$

- Denote $P_t^R(n)$ as the real bond price, with a payoff of inflation, $\Pi(t+n)/\Pi(t)$, where $\Pi(t)$ is the price level at time t. $y_t^R(n)$ is the corresponding bond yield.

- The nominal **forward rate** is defined as

$$f_t(n) = \log P_t(n-1) - \log P_t(n) = p_t(n-1) - p_t(n),$$

  which is the rate at which you can invest between $t+n-1$ and $t+n$.

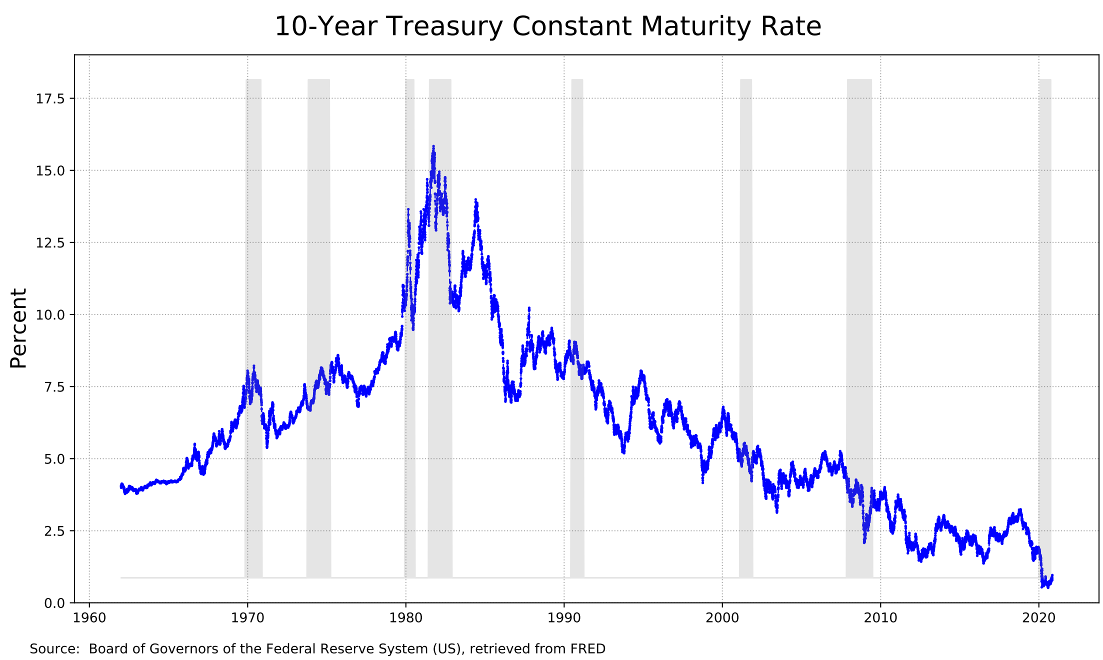
- Log excess **holding period returns** on a n-period zero coupon bond:

$$rx_{t+1}(n) = p_{t+1}(n-1) - p_t(n) - y_t(1).$$

2.2. *Facts*

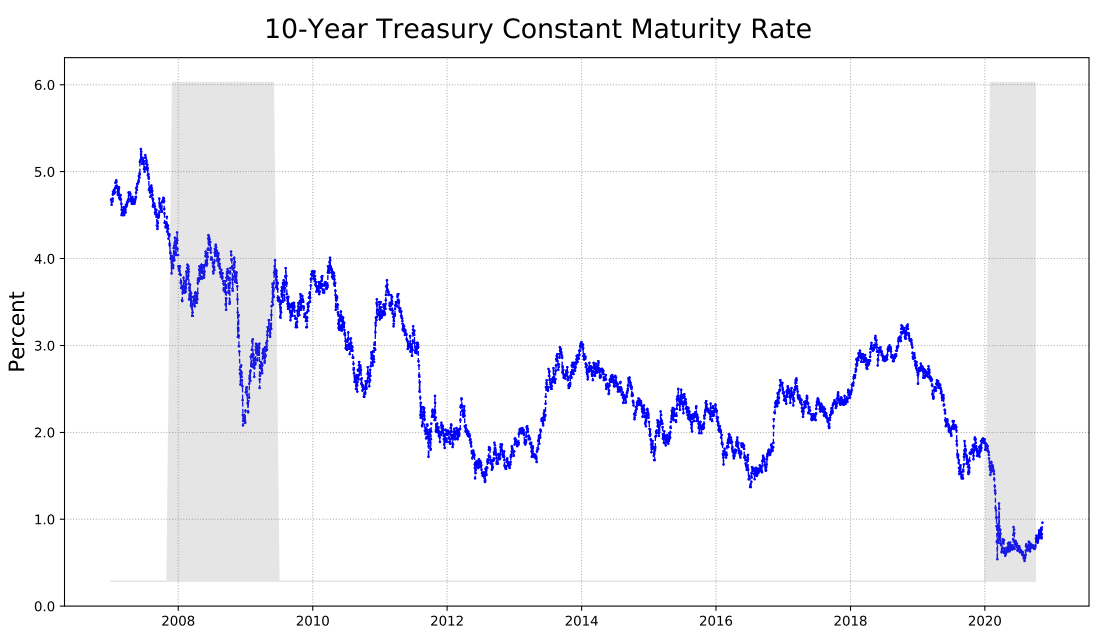
2.2.1. The dynamics of bond yields

- U.S. bond 10-year constant maturity bond yield from Jan 1962-Nov 2020:



- Observations:
  - Low-frequency dynamics in bond yields, connected to high inflation in the late 1970s and early 1980s and subsequent reversal.
  - This reversal is usually attributed to the Central Bank regaining control over inflation by re-establishing credibility (Clarida, Gali, and Gertler (1999)).
  - Recent work by Drechsler, Savov, and Schnabl (2020) instead ascribes it to the repeal of Regulation Q which set ceilings on deposit interest rates.

- Business cycle variation in yields around the trend. Fed raises Federal Funds rate in expansions to prevent the economy from overheating and inflation from ramping up. Lowers rates in recessions or to stave off a recession.
- The current low-rate environment may not be as surprising given the long-term pattern in declining yields.
- In 2018, investors worried that the 30-year bull market in bonds had come to an end. The 10-year yield rose from 1.4% in July 2016 to 3.2% in October 2018.
- But October 2018 turned out to be a local maximum; 10-year yield fell back to 1.5% in August 2019.
- With the arrival of the covid-19 pandemic, the Fed slashed the FFR twice in March 2020 by total of 150 bps. The 10-year yield fell to 0.60%, a new historical record low.
- Since then, 10-year yield has gone back up to 1.6%.

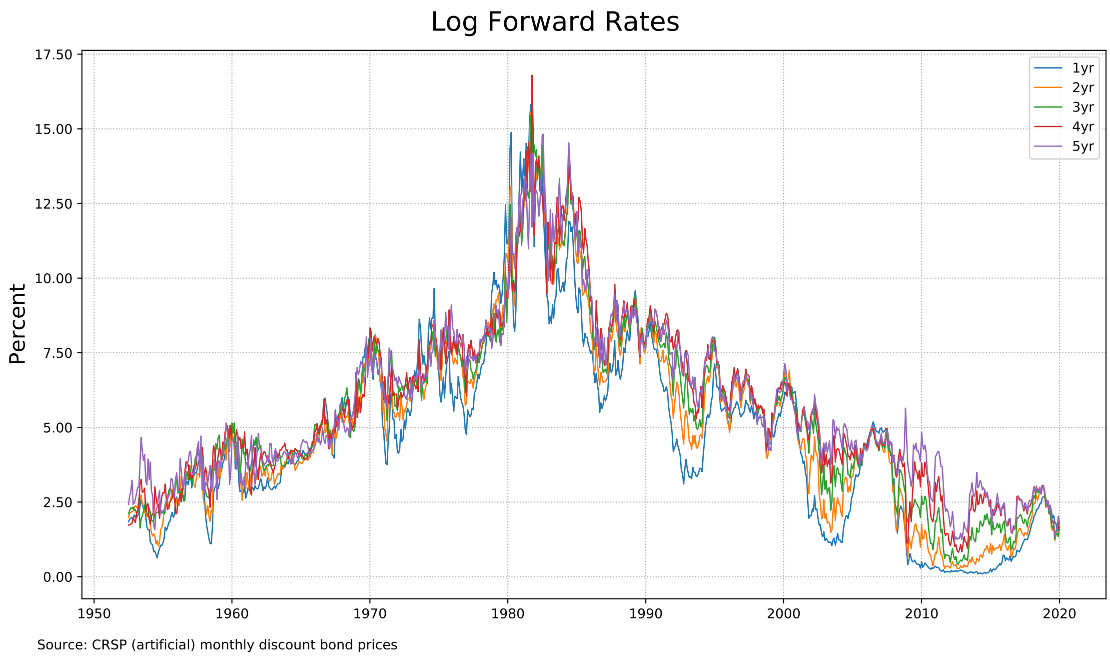


Source: Board of Governors of the Federal Reserve System (US), retrieved from FRED

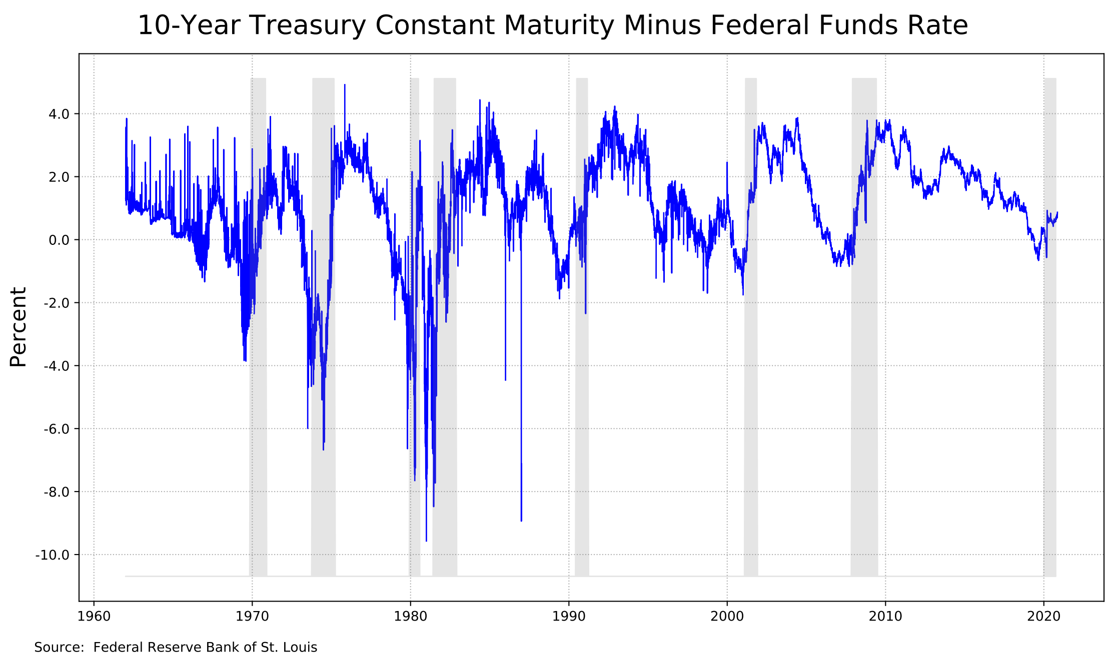
- Log forward rates $f_t(n) = p_t(n-1) - p_t(n)$ on 1- through 5-year yields. Monthly Fama-Bliss data (CRSP) data for 1952.6-2019.12.

- Forward rates show similar persistence as bond prices/yields

- Forward rates decompose bond yields into the various horizon contributions.

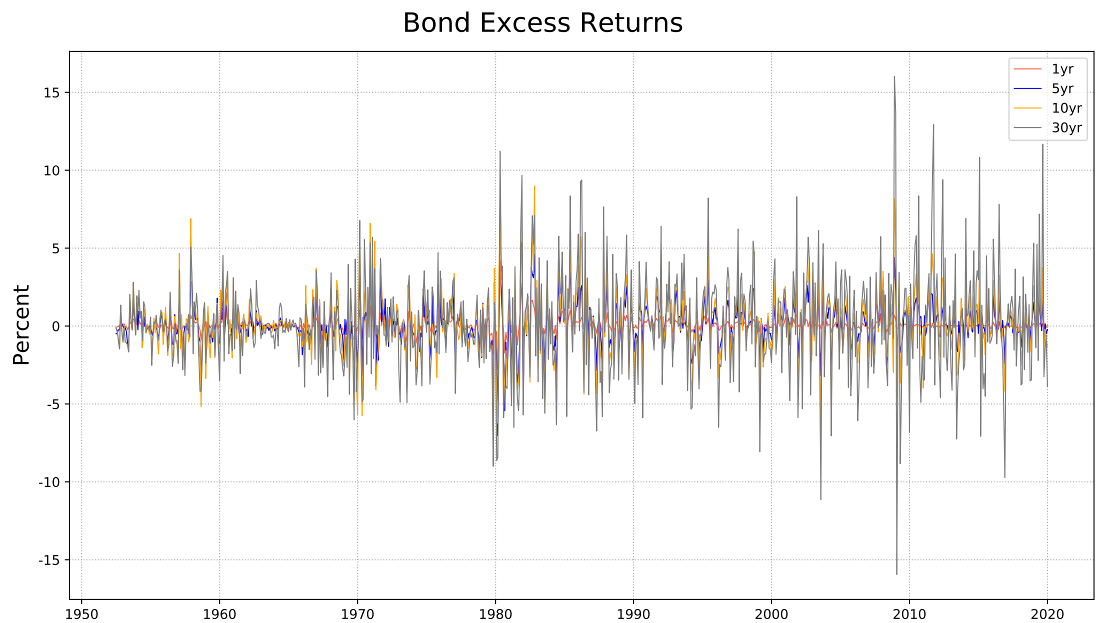


- To remove the low-frequency component in yields, we can look at the yield spread. Here, the difference between the 10-year constant maturity Treasury yield and the federal funds rate.



  - The yield spread is low, usually negative, at the onset of recessions.
  - The yield spread rises during recessions.
  - What is the economic interpretation?
  - U.S. recessions are dated by the NBER. The ECRI follows a similar methodology internationally. Note that recessions are dated ex-post, not in real time. The latest recession began in February 2020 and ended in April 2020, the shortest recession on record.
  - Slope of yield curve fell from 2.0% in December 2016 to -0.6% in August 2019. Now back to 1.4%.
  - Yield curve inversions have predicted 11 of the last 8 recessions!

- Bond excess returns $r_t(n) - y_t(1)$, where $r_t(n) = p_{t+1}(n-1) - p_t(n)$, is the log monthly holding period return in excess of the one-month yield, multiplied by 100.



n (years)	0.25	1	2	3	4	5
$\mathbb{E}[y_t(n)]$	4.34	4.69	4.89	5.07	5.22	5.33
$Std[y_t(n)]$	3.14	3.18	3.14	3.07	3.02	2.95
$AC(12)[y_t(n)]$	0.87	0.88	0.90	0.91	0.92	0.93
$\mathbb{E}[y_t(n) - y_t(0.25)]$	–	0.35	0.55	0.73	0.88	0.99
$Std[y_t(n) - y_t(0.25)]$	–	0.39	0.61	0.77	0.89	0.98
$AC(12)[y_t(n) - y_t(0.25)]$	–	0.16	0.25	0.31	0.37	0.39

TABLE 1
SUMMARY STATS ON YIELD LEVELS AND SLOPES

n (years)	1	2	5	7	10	20	30
$\mathbb{E}[r_t(n) - y_t(1/12)]$	0.91	1.13	1.57	1.85	1.51	1.92	1.69
$Std[r_t(n)]$	1.75	2.71	4.94	6.04	7.23	9.56	10.89
$SR[r_t(n)]$	0.52	0.42	0.32	0.31	0.21	0.20	0.15

TABLE 2
SUMMARY STATS ON EXCESS RETURNS

2.2.2. Factor structure in yields

- Bond yields have a strong factor structure across maturities.
- Use principal components analysis (PCA) to show this.
- Denote the covariance matrix of N bond yields by $\Sigma = Var(y_t)$.
- The first principal component is a linear combination of yields that has maximum variance,

$$\max_w w'\Sigma w,$$

  such that $w'w = 1$.

- The second principal component (factor) is found by maximizing the residual variance and making sure that the second component is orthogonal to the first component.
- You can find that factor by computing the eigenvalue decomposition of the covariance matrix of bond yields

$$\Sigma = Q\Lambda Q',$$

  where the columns Q correspond to the eigenvectors and the diagonal matrix Λ contains the eigenvalues.

- Assuming the eigenvalues are ordered from largest to smallest ($\Lambda_{1,1}$ is the largest), then $Q'_{(:,n)}y_t$ is the n^{th} factor. The fraction of variance explained by this factor is

$$\Lambda_n/\iota'\Lambda\iota.$$

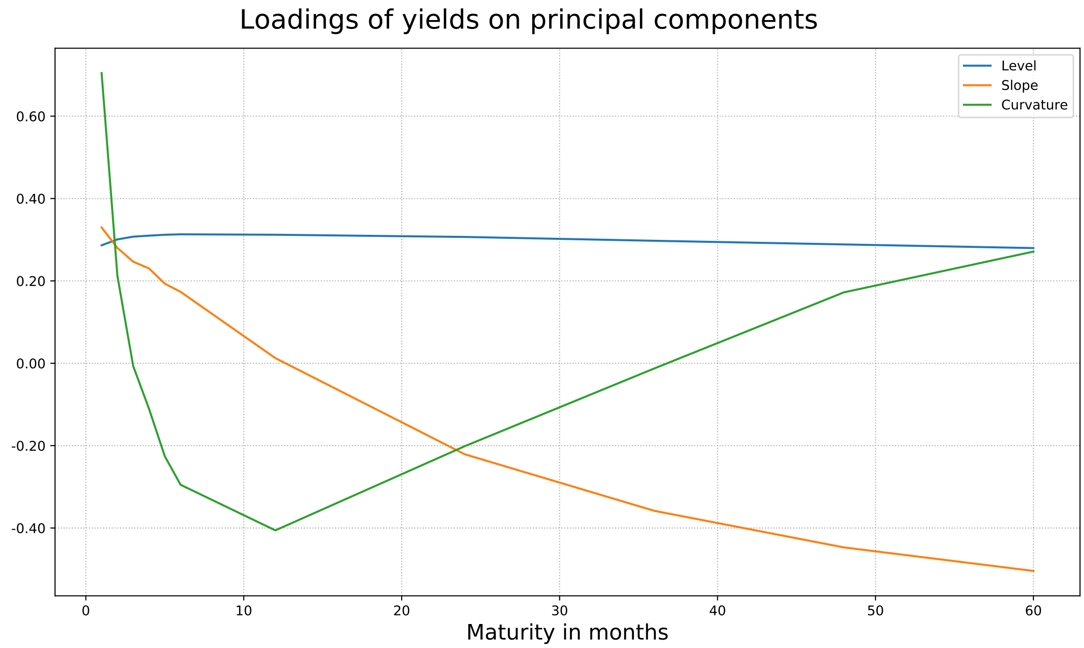
- Based on Piazzesi (2010) and updated for sample period 1964/01 - 2019/12:

	PC1	PC2	PC3	PC4	PC5
% variance explained Y_t	0.9824	0.9978	0.9991	0.9996	0.9997
% variance explained ΔY_t	0.7829	0.9108	0.9620	0.9734	0.9820

TABLE 3

- In levels, a single PC explains 98.2% of the variation in yields.
- This is driven by the low-frequency component in yields, the "level factor."
- Three PCs explain 99.9% of the yield variation.
- Even in changes, which removes a big chunk of the low-frequency component, there is a very strong factor structure.

- The loadings of yields on the first three principal components:



- The fact that you get a level, slope, and a curvature factor may not contain a lot of economics, see Lord and Pelsser (2007).

- In any case, a low-dimensional factor model suffices to explain most of the variation in yields.

2.2.3. Risk premia and Sharpe ratios across maturities

- Discounting across maturities plays a central role in asset pricing and corporate finance.
- For instance, the how to value an investment project or a private equity firm?
- Hence, we need to measure discount rates across maturities.
- Average returns, volatilities, and Sharpe ratios on Treasuries sorted by maturity bucket from Binsbergen and Koijen (2017)

	1-12	13-24	25-36	37-48	49-60	61-120
Average excess return	0.58%	1.03%	1.36%	1.56%	1.56%	1.83%
Standard deviation	0.80%	2.05%	3.13%	3.95%	4.67%	5.76%
Sharpe ratio	0.73	0.50	0.43	0.40	0.33	0.32

Table 4: We summarize the annualized average excess return, standard deviation and Sharpe ratios of nominal Treasury bond returns. The maturities (in months) are summarized in the first row of the table. The sample period is from January 1952 until December 2013.

- The maturity buckets are in months.
- Observations:
  - Average returns increase with maturity.
  - Volatilities increase with maturity as well.
  - Sharpe ratios decline very rapidly with maturity. Downward sloping term structure of T-bond strips on Sharpe ratios consistent with facts on dividend strips
- Sharpe ratios of short-term bonds are very high, compared to for instance equity markets.
- See also Hansen and Jagannathan (1991) and Luttmer (1996).

2.2.4. Time-series Predictability

- Standard term structure models (more later) imply that information about bond risk premia is embedded in bond yields or forward rates.

- Starting point of his literature is the (generalized) **expectation hypothesis**. Three statements of the EH:

  1. The yield of a bond with maturity n is equal to the average of the expected yields of future one-year bonds (up to a constant risk premium):

  $$y_t(n) = \frac{1}{n} E_t \left[y_t(1) + y_{t+1}(1) + ... + y_{t+n-1}(1) \right] + \text{ (risk premium)}.$$

  2. The forward rate equals the expected future spot rate (up to a constant risk premium):

  $$f_t(n) = E_t \left[y_{t+n-1}(1) \right] + \text{ (risk premium)}.$$

  3. The expected holding-period return is the same for any bond maturity n (up to a constant risk premium):

  $$E_t[r_{t+1}(n)] = y_t(1) + \text{ (risk premium)}, \forall n.$$

- These three definitions are equivalent. (The risk premium terms are different in the three statements of the generalized EH.)

- For example, start from $f_t(n) = E_t[y_{t+n-1}(1)]$

- Add these up over n periods to obtain:

$$\begin{aligned}
E_t(y_t(1) + y_{t+1}(1) + ... + y_{t+n-1}(1)) &= f_t(1) + f_t(2) + ... + f_t(n) \\
&= (p_t(0) - p_t(1)) + (p_t(1) - p_t(2)) + ... \\
&\quad + (p_t(n-1) - p_t(n)) \\
&= -p_t(n) \\
&= ny_t(n)
\end{aligned}$$

which recovers that long yields equal average expected future short rates.

- From the definition of returns and yields:

$$\begin{aligned}
y_t(2) - y_t(1) &= \frac{1}{2}(r_{t+1}(2) - y_t(1) + r_{t+2}(1) - y_t(1)) \\
&= \frac{1}{2}(2y_t(2) - y_{t+1}(1) - y_t(1) + y_{t+1}(1) - y_t(1))
\end{aligned}$$

where we used the definition of a bond return:
$r_{t+1}(n) = p_{t+1}(n-1) - p_t(n) = -(n-1)y_{t+1}(n-1) + ny_t(n)$

- Similarly, for generic maturity n:

$$\begin{aligned}
y_t(n) - y_t(1) &= \frac{1}{n}\sum_{i=0}^{n-1}(r_{t+i+1}(n-i) - y_t(1)) \\
&= \frac{1}{n}\sum_{i=0}^{n-1}\left[(y_{t+i}(1) - y_t(1)) + (r_{t+i+1}(n-i) - y_{t+i}(1))\right] \\
&= \frac{1}{n}\sum_{i=0}^{n-1}E_t\left[y_{t+i}(1) - y_t(1)\right] + \frac{1}{n}\sum_{i=0}^{n-1}E_t\left[r_{t+i+1}(n-i) - y_{t+i}(1)\right]
\end{aligned}$$

- Last term is an expected excess return = bond risk premium

- Bond risk premium it is zero (constant) under the (generalized) expectations hypothesis

- Under the EH, the slope of the yield curve is the average expected change in future short rates:

$$y_t(n) - y_t(1) = \frac{1}{n}\sum_{i=0}^{n-1} E_t\left[y_{t+i}(1) - y_t(1)\right] = \frac{1}{n}\sum_{i=0}^{n-1} E_t\left[(n-i)\Delta y_{t+i}(1)\right]$$

- Can test this by running forecasting regression of changes in future realized short rates on the lagged yield spread:

$$\frac{1}{n}\sum_{i=1}^{n-1}(n-i)\,\Delta y_{t+i}(1) = \gamma_{n,0} + \gamma_{n,1}(y_t(n) - y_t(1)) + \varepsilon_t$$

  EH predicts that $\gamma_{n,1} = 1, \forall n$

- Result using monthly Fama-Bliss zero-coupon data from CRSP for 1952.6-2019.12. Newey-West t-statistics are reported in brackets; 12 lags used because overlapping monthly data.

n (years)	2	3	4	5
$\gamma_{n,1}$	0.24	0.41	0.56	0.68
$t-stat$	[3.46]	[2.27]	[1.87]	[1.54]
R^2	1.06%	3.42%	7.21%	11.39%

- EH fails: coefficients $\gamma_{n,1}$ significantly smaller than one

- The yield spread does forecast future short rate changes, but the subsequent changes in short rates are too small to enforce expectations hypothesis

- As maturity increases, coefficients become closer to one.

- Results imply that bond risk premia must be time-varying by the slope of the yield curve.

- Campbell and Shiller (1991) show that the yield spread forecasts future excess bond returns. They estimate the **bond return predictability** equation:

$$\frac{1}{n}\sum_{i=0}^{n-1} r_{t+i+1}(n-i) - y_{t+i}(1) = \gamma_{n,0} + \gamma_{n,1}(y_t(n) - y_t(1)) + \varepsilon_t$$

  EH predicts that $\gamma_{n,1} = 0$, $\forall n$

- We update CS's result using data from 1952.6-2019.12:

n (years)	2	3	4	5
$\gamma_{n,1}$	0.76	0.59	0.44	0.32
$t - stat$	[3.46]	[2.27]	[1.87]	[1.54]
R^2	9.50%	6.86%	4.59%	2.87%

- If the expectations hypothesis holds for interest rates, then the forward rate equals the expected future spot rate:

$$f_t(n) = E_t[y_{t+n-1}(1)]$$

- In post-war data, all interest rates and forward rates share a very persistent component due to inflation. This makes yields a near-unit root processes. Better to take out the short rate.

- Fama and Bliss (1987) posit the following linear regression model for the long-run change in short rates:

$$y_{t+n-1}(1) - y_t(1) = a_{n,0} + a_{n,1}\left(f_t(n) - y_t(1)\right) + \varepsilon_{t+n-1},\ n = 1, 2, 3, 4$$

  EH predicts that $a_{n,1} = 1$, $\forall n$

- Results with monthly Fama-Bliss zero-coupon data from CRSP 1952.6-2019.12. Newey-West standard errors are reported in brackets; 12 lags were used because overlapping monthly data.

n (years)	2	3	4	5
$\alpha_{n,1}$	0.24	0.54	0.72	0.78
$t - stat$	[-3.46]	[-1.57]	[-1.34]	[-1.18]
R^2	1.06%	5.50%	11.50%	14.66%

- EH fails: the $a_{n,1}$ slope coefficients are too small

- Changes in forward rates do not translate one-for-one into changes in short yields. The subsequent changes in short yields are too small relative to what is predicted by the expectations hypothesis.

- Again, the flip side of this is that forward rates should predict returns! This is what Fama and Bliss indeed find.
- Lets run the following **bond return predictability** regression

$$r_{t+1}(n) - y_t(1) = \gamma_{n,0} + \gamma_{n,1}\left(f_t(n) - y_t(1)\right) + \varepsilon_{t+1},\ n = 1, 2, 3, 4$$

  and test $H_0 : \gamma_{n,1} = 0,\ \forall n$

- Result with monthly Fama-Bliss zero-coupon data from CRSP for 1952.6-2019.12. Newey West t-statistics are reported in brackets; 12 lags were used to compute standard errors.

n (years)	2	3	4	5
$\gamma_{n,1}$	0.76	1.00	1.27	1.06
$t-stat$	[3.46]	[3.59]	[4.03]	[3.00]
R^2	9.50%	10.63%	13.58%	7.88%

- Indeed, there is strong evidence that the forward spread predicts future bond returns, as suggested by the failure of the EH.

- Cochrane and Piazzesi (2005) 's idea: why not use all forward rates to predict excess returns?
- First, they regress bond excess returns of different horizons $n = 1, \cdots, 5$ on all lagged forward rates:

$$r_{t+1}(n) - y_t(1) = \alpha_n + \beta_n^1 f_t(1) + \beta_n^2 f_t(2) + \beta_n^3 f_t(3) + \beta_n^4 f_t(4) + \beta_n^5 f_t(5) + \varepsilon_{t+1}$$

- Cross-sectional average return: $\overline{rx}_{t+1} = 0.25 \times \sum_{n=2}^{5} r_{t+1}(n) - y_t(1)$.

$$\overline{rx}_{t+1}(n) = \gamma_0 + \gamma_1 f_t(1) + \gamma_2 f_t(2) + \gamma_3 f_t(3) + \gamma_4 f_t(4) + \gamma_5 f_t(5) + \varepsilon_{t+1}$$

- Define the CP factor as the fitted value of this regression:

$$CP_t = \hat{\gamma}_0 + \hat{\gamma}' f_t.$$

- This proxy for the bond risk premium does a good job forecasting each and every bond excess return at various horizons:

$$rx_{t+1}(n) \;=\; b_n \left(\gamma_0 + \sum_{n=1}^{5} \gamma_n f_t(n) \right) + \epsilon_{t+1}(n),$$

  hence there is a **common factor** that predicts all excess returns.

- Main regression tables:

TABLE 1—ESTIMATES OF THE SINGLE-FACTOR MODEL

A. Estimates of the return-forecasting factor, $\overline{rx}_{t+1} = \gamma^\top \mathbf{f}_t + \bar{\varepsilon}_{t+1}$

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	R^2	$\chi^2(5)$
OLS estimates	−3.24	−2.14	0.81	3.00	0.80	−2.08	0.35	
Asymptotic (Large T) distributions								
HH, 12 lags	(1.45)	(0.36)	(0.74)	(0.50)	(0.45)	(0.34)		811.3
NW, 18 lags	(1.31)	(0.34)	(0.69)	(0.55)	(0.46)	(0.41)		105.5
Simplified HH	(1.80)	(0.59)	(1.04)	(0.78)	(0.62)	(0.55)		42.4
No overlap	(1.83)	(0.84)	(1.69)	(1.69)	(1.21)	(1.06)		22.6
Small-sample (Small T) distributions								
12 lag VAR	(1.72)	(0.60)	(1.00)	(0.80)	(0.60)	(0.58)	[0.22, 0.56]	40.2
Cointegrated VAR	(1.88)	(0.63)	(1.05)	(0.80)	(0.60)	(0.58)	[0.18, 0.51]	38.1
Exp. Hypo.							[0.00, 0.17]	

B. Individual-bond regressions

	Restricted, $rx_{t+1}^{(n)} = b_n(\gamma^\top \mathbf{f}_t) + \varepsilon_{t+1}^{(n)}$					Unrestricted, $rx_{t+1}^{(n)} = \beta_n \mathbf{f}_t + \varepsilon_{t+1}^{(n)}$			
n	b_n	Large T	Small T	R^2	Small T	R^2	EH	Level R^2	$\chi^2(5)$
2	0.47	(0.03)	(0.02)	0.31	[0.18, 0.52]	0.32	[0, 0.17]	0.36	121.8
3	0.87	(0.02)	(0.02)	0.34	[0.21, 0.54]	0.34	[0, 0.17]	0.36	113.8
4	1.24	(0.01)	(0.02)	0.37	[0.24, 0.57]	0.37	[0, 0.17]	0.39	115.7
5	1.43	(0.04)	(0.03)	0.34	[0.21, 0.55]	0.35	[0, 0.17]	0.36	88.2

Notes: The 10-percent, 5-percent and 1-percent critical values for a $\chi^2(5)$ are 9.2, 11.1, and 15.1 respectively. All p-values are less than 0.005. Standard errors in parentheses "( )", 95-percent confidence intervals for R^2 in square brackets "[ ]". Monthly observations of annual returns, 1964–2003.

TABLE 3—FORECASTS OF EXCESS STOCK RETURNS

Right-hand variables		$\gamma^\top \mathbf{f}$	(t-stat)	d/p	(t-stat)	$y^{(5)} - y^{(1)}$	(t-stat)	R^2
1	$\gamma^\top \mathbf{f}$	1.73	(2.20)					0.07
2	D/p			3.30	(1.68)			0.05
3	Term spread					2.84	(1.14)	0.02
4	D/p and term			3.56	(1.80)	3.29	(1.48)	0.08
5	$\gamma^\top \mathbf{f}$ and term	1.87	(2.38)			−0.58	(−0.20)	0.07
6	$\gamma^\top \mathbf{f}$ and d/p	1.49	(2.17)	2.64	(1.39)			0.10
7	All $\mathbf{f}$							0.10
8	Moving average $\gamma^\top \mathbf{f}$	2.11	(3.39)					0.12
9	MA $\gamma^\top \mathbf{f}$, term, d/p	2.23	(3.86)	1.95	(1.02)	−1.41	(−0.63)	0.15

Notes: The left-hand variable is the one-year return on the value-weighted NYSE stock return, less the 1-year bond yield. Standard errors use the Hansen-Hodrick correction.

- Main disadvantage of this approach is that we regress price changes on prices, and we do not quite understand the economic drivers of risk premia.

- Cochrane and Piazzesi suggest a link to business cycles; more on this later.

- Ludvigson and Ng (2009) make progress on linking bond risk premia to macro-economic fundamentals.

- They use factor analysis on many macro-economic time series to extract factors. First 8 principal components explain about 50% of variation in macro series. They then explore which factors predict bond returns, alongside CP, (for 2-year and 5-year bonds):

Table 2
Regression of monthly excess bond returns on lagged cactors

Model: $rx^{(n)}_{t+1} = \beta_0 + \beta_1' \widehat{F}_t + \beta_2 CP_t + \epsilon_{t+1}$,

		$\widehat{F}_{1t}$	$\widehat{F}^3_{1t}$	$\widehat{F}_{2t}$	$\widehat{F}_{3t}$	$\widehat{F}_{4t}$	$\widehat{F}_{8t}$	CP_t	$F5_t$	$F6_t$	$\bar{R}^2$
$rx^{(2)}_{t+1}$	(a)							**0.45** (8.90)			0.31
	(b)	**−0.93** (−5.19)	**0.06** (2.78)	**−0.40** (−3.10)	**0.18** (2.24)	**−0.33** (−2.94)	**0.35** (4.35)				0.26
	(c)	**−0.74** (−4.48)	**0.05** (2.70)	0.08 (0.71)	**0.24** (3.84)	**−0.24** (−2.51)	**0.24** (2.70)	**0.41** (5.22)			0.45
	(d)	**−0.93** (−4.96)	**0.06** (2.87)		0.18 (1.87)	**−0.33** (−2.65)	**0.35** (3.83)				0.22
	(e)	**−0.75** (−4.71)	**0.05** (2.71)		**0.24** (3.85)	**−0.25** (−2.61)	**0.24** (2.89)	**0.40** (5.89)			0.45
	(f)								**0.54** (5.52)		0.22
	(g)									**0.50** (6.78)	0.26
	(h)							**0.39** (6.0)	**0.43** (5.78)		0.44
$rx^{(5)}_{t+1}$	(a)							**1.46** (7.90)			0.34
	(b)	**−2.27** (−4.10)	**0.18** (3.06)		0.18 (0.55)	−0.78 (−1.80)	**1.13** (3.68)				0.14
	(c)	**−1.63** (−3.86)	**0.15** (2.95)		**0.38** (1.92)	−0.48 (−1.54)	**0.76** (2.76)	**1.34** (6.00)			0.41
	(d)								**1.36** (4.80)		0.14
	(e)									**1.41** (6.47)	0.21
	(f)							**1.32** (5.87)	**0.98** (5.08)		0.42

- F5 (F6) is the single linear combination of 5 (6) factors $\hat{F}$ that best predicts the average excess return of maturities 2-5.

- Best macro factor (F6) does about 2/3 as well as CP in forecasting future bond returns in isolation. F5 and CP both enter strongly significantly together, leading to 40% R^2.

- They identify real ($\hat{F}_1$, $\hat{F}_5$) and inflation ($\hat{F}_3$, $\hat{F}_4$) factors, which have predictive power *beyond forward rates and yield factors*. This has important implication for affine term structure models, which we will return to below.

- Estimated bond risk premia are counter-cyclical (correlation of F5 with industrial production growth is -71%):

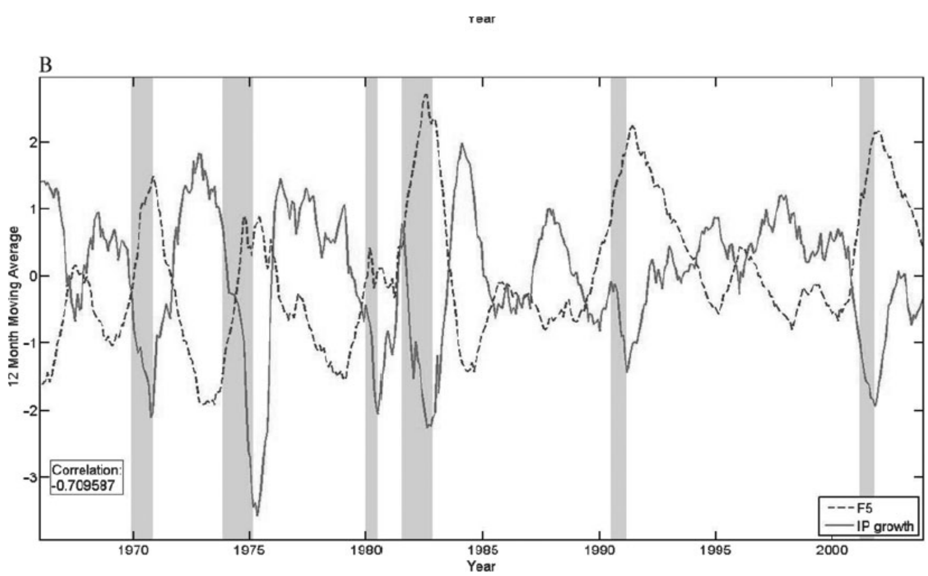


Figure 6
A: First factor and IP growth. B: *F*5 and IP growth
Note: Standardized units are reported. Shadings denote months designated as recessions by the National Bureau of Economic Research. "First factor" denotes the first estimated factor, F_1t. $F5$ denotes the linear combination of five factors, written in the text as $F5_t$.

- Bianchi, Büchner, and Tamoni (2021) use machine learning techniques (esp. neural networks) to study the best combination of yields and macro-economic information to predict excess bond returns. Data is 1981-2018, with OOS period starting in 1990.

- Non-linear combinations of yields predict excess returns out of sample using simple (shallow) neural networks: $R^2_{oos} = 5\%$ for EW bond excess return for the EW average of NN model specifications. Linear models (PCA, PLS) do **not**.

- Adding the information from macro-economic series improves the forecast relative to the model that just uses yields: $R^2_{oos} = 20\%$ for EW bond excess return.

- An investor forming a portfolio of 2- to5-, 7-, and 10-year bonds generates a 3.5% point higher annual return when using the NN model with macro variables in the portfolio formation than when using the best NN model estimated with yield data only.

- A puzzling feature of the Cochrane-Piazzesi factor is that it is orthogonal to the first three principal components of yields.

- Cieslak and Povala (2015) start from the basic decomposition of yields into expectations of future short rates and risk premia

$$y_t(n) = \frac{1}{n}\sum_{i=0}^{n-1} E_t\left[y_{t+i}(1)\right] + rpy_t(n).$$

- Recall that under the expectations hypothesis, $rpy_t(n) = 0$

- Short rates contain two components, expected inflation (τ_t) and the real rate ($y_t^R(1)$),

$$y_t(1) = \tau_t + y_t^R(1).$$

- Expected inflation ("trend inflation") is highly persistent.

- Trend inflation is measured using a moving average of past monthly inflation over last 10 years ($\nu = .987$):

$$\tau_t = (1-\nu)\sum_{s=0}^{t-1} \nu^s \pi_{t-s}, \quad \pi_t = \ln\left(\frac{CPI_t}{CPI_{t-1}}\right).$$

- The low-frequency component of yields relates to inflation expectations (underlying the Federal Reserve Board's FRB/US model).

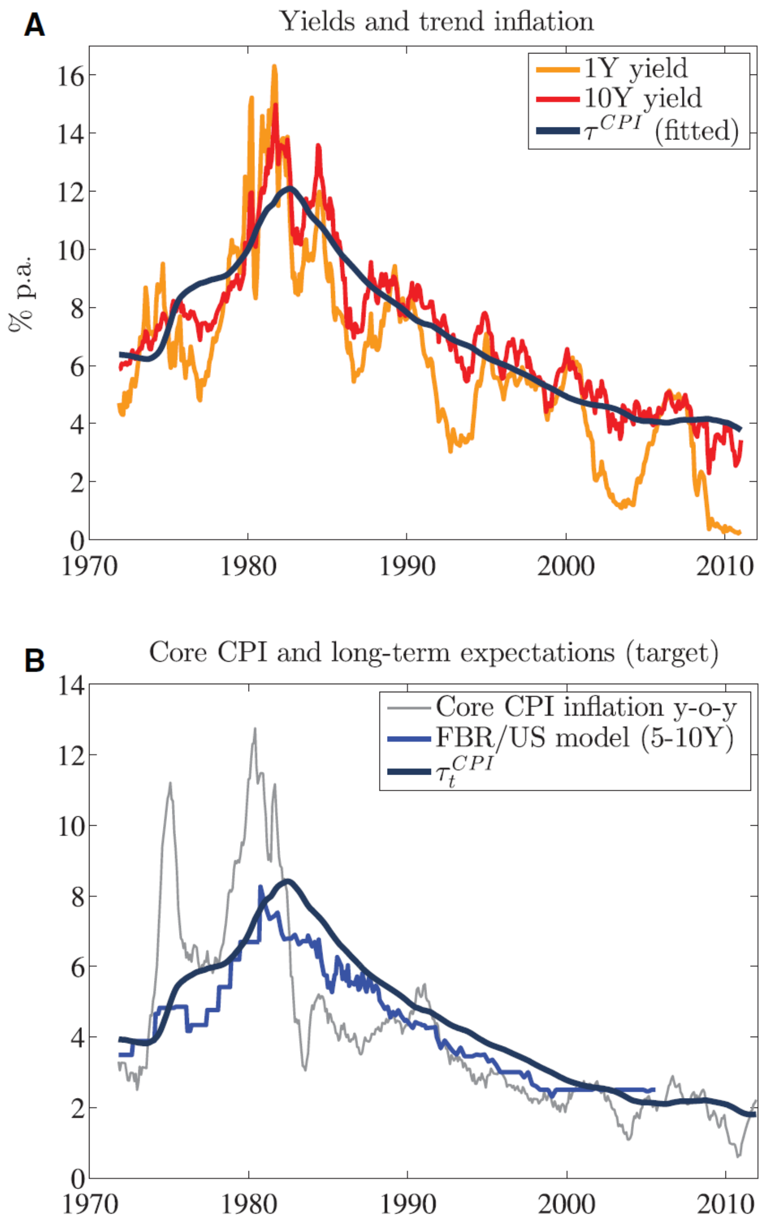


Figure 1
Measuring trend inflation, τ_t^{CPI}
Panel A superimposes the 1- and 10-year yield with the DMA of past core CPI inflation, τ_t^{CPI}. τ_t^{CPI} is fitted to the average level of yields across maturities (slope coefficient of 1.28). Panel B plots the realized year-on-year core CPI inflation together with τ_t^{CPI}, and the long-term (5–10 years ahead) inflation expectations used in the Federal Reserve Board's FRB/US model.

- Next, they decompose each yield into trend and cycle

$$y_t(n) = a_n + b_n \tau_t + \epsilon_t,$$

  where we define the cycle component as $c_t(n) = \widehat{\epsilon}_t$.

- Lastly, much like Cochrane and Piazzesi, they consider the forecasting regression

$$\overline{rx}_{t+1} = \gamma_0 + \gamma_1 \overline{c}_t + \gamma_2 c_t(1) + u_{t+1},$$

  where $\overline{c}_t = \frac{1}{N-1} \sum_{i=2}^{N} c_t(i)$.

- The "cycle" factor is defined as

$$\widehat{cf}_t = \widehat{\gamma}_0 + \widehat{\gamma}_1 \overline{c}_t + \widehat{\gamma}_2 c_t(1).$$

- Economically, we will see that $c_t(1)$ corresponds to the short-term real interest rate and $\overline{c}_t$ is the risk premium component of yields.

- This leads to a natural decomposition of yields into inflation expectations, the real rate, and a risk premium component.

- Predictive regressions

Table 2
Predictive regressions

A. Predictive regressions

Regressors →	Yields only (1)	Yields+τ^{CPI} (2)	$\bar{y}_t, y_t^{(1)}$ (3)	$\bar{y}_t, y_t^{(1)}, \tau_t^{CPI}$ (4)	$\bar{c}_t, c_t^{(1)}$ (5)
Regression coefficients					
$y^{(1)}$ or $c^{(1)}$	−1.13	−1.09	−0.42	−0.61	−0.61
	(−1.87)	(−1.64)	(−2.48)	(−3.70)	(−3.67)
$y^{(2)}$ or $c^{(2)}$	0.73	1.06	–	–	–
	(0.62)	(0.81)	–	–	–
$y^{(5)}$ or $c^{(5)}$	0.83	−0.71	–	–	–
	(0.99)	(−0.10)	–	–	–
$y^{(7)}$ or $c^{(7)}$	0.40	0.51	–	–	–
	(0.15)	(0.32)	–	–	–
$y^{(10)}$ or $c^{(10)}$	−1.15	0.84	–	–	–
	(−1.69)	(0.43)	–	–	–
$y^{(20)}$ or $c^{(20)}$	0.37	0.21	–	–	–
	(0.94)	(0.49)	–	–	–
τ^{CPI}	–	−1.02	–	−1.01	–
	–	(−4.30)	–	(−4.65)	–
$\bar{y}$ or $\bar{c}$	–	–	0.54	1.45	1.45
	–	–	(2.47)	(5.03)	(5.03)
Regression statistics					
$\bar{R}^2$	0.24	0.54	0.18	0.53	0.53
Wald test	12.34	34.86	6.46	28.61	25.34
pval	0.05	0.00	0.04	0.00	0.00
Rel.prob. (BIC)	0	3e-4	0	0.57	1.00

B. Distribution of predictive $\bar{R}^2$ under EH, $T = 470$ months

	$\phi_r = 0.75$			$\phi_\tau = 0.975$		
	$\phi_\tau = 0.8$	$\phi_\tau = 0.975$	$\phi_\tau = 0.999$	$\phi_r = 0.6$	$\phi_r = 0.75$	$\phi_r = 0.9$
P5	0.00	0.01	0.01	0.01	0.01	0.01
P95	0.19	0.23	0.20	0.22	0.22	0.23

In panel A, the LHS variable is a duration-standardized excess bond return averaged across maturities, $\overline{rx}_{t+1}$. Columns (1) through (5) use different regressors: (1) six yields; (2) same yields as in (1) plus trend inflation τ_t^{CPI}; (3) two yield variables: $y_t^{(1)}$ and $\bar{y}_t$; (4) $y_t^{(1)}$ and $\bar{y}_t$ plus τ_t^{CPI}; (5) two cycle variables: $c_t^{(1)}$ and the average cycle $\bar{c}_t$. T-statistics for individual coefficients, the Wald test and the corresponding p-values are obtained using the reverse regression delta method. Row labeled "Rel.prob. (BIC)", where BIC is the Bayesian information criterion, gives the relative probability of a model i computed as $\exp\{(BIC_{\text{best}} - BIC_i)T/2\}$, where $BIC = \ln(\widehat{\sigma^2}) + \ln(T)n/T$, n is the number of regressors, $\widehat{\sigma^2} = SSE/T$ of the regression, and T is the sample size. Relative probability of one indicates the best model selected by a given criterion. Relative probability of zero means that a given model has zero probability to explain the data equally well as the best model. Panel B reports the 5th and 95th percentiles of the $\bar{R}^2$ obtained under the null of EH from 10,000 Monte Carlo simulations of the model in Section 1. The parameters are $\delta_0 = 0, \delta_\tau = 1.43, \delta_r = 1$, and σ_τ, σ_r are calibrated to match st.dev.$(\tau_t) = 1.90\%$ and st.dev.$(r_t) = 1.74\%$ at each level of persistence of ϕ_τ, ϕ_r.

- The R-squared of the real rate factor and the risk premium factor is very high, $R^2 = 53\%$.

  This implies that the excess return predictability in the bond market is much stronger than in equity markets.

- Most of the variation in the cycle factor, the estimate of bond risk premia, is driven by $\bar{c}_t$

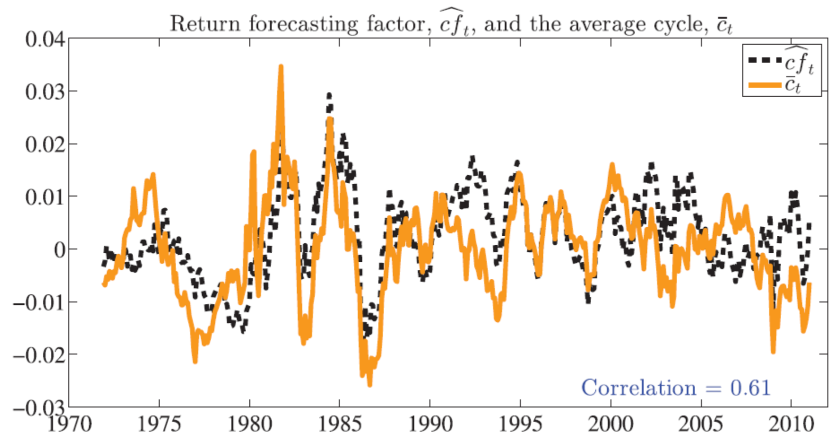


Figure 2
The cycle factor $\widehat{cf}$ and the average cycle, $\bar{c}_t$
Figure 2 shows the time series of the cycle factor $\widehat{cf}_t$ and the average cycle across maturities $\bar{c}_t$.

- $c_t(1)$ is closely connected to other measures of the real rate

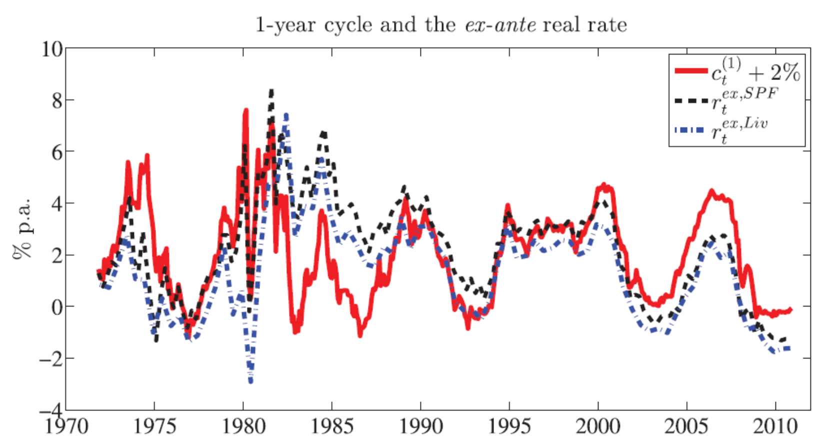


Figure 3
Short-maturity cycle and the *ex-ante* real rate
The figure compares the *ex-ante* real rate with 1-year interest rate cycle, $c_t^{(1)}$. The *ex-ante* real rate is obtained as $r_t^{ex,s} = y_t^{(1)} - E_t^s(\pi_{t+1})$, where $E_t^s(\pi_{t+1})$ is expected inflation 1-year ahead from Livingston $s = Liv$ or SPF $s = SPF$ survey, respectively. For ease of comparison, we add 2% to $c_t^{(1)}$. The Livingston survey is available semiannually, and the SPF survey is quarterly.

- Decomposition of yields in terms of the three factors
  - Expected inflation - strong correlation with *Level* factor
  - Real rate - strong correlation with *Slope* factor
  - Risk premium - increasing contribution with maturity, also correlated with *Slope* factor

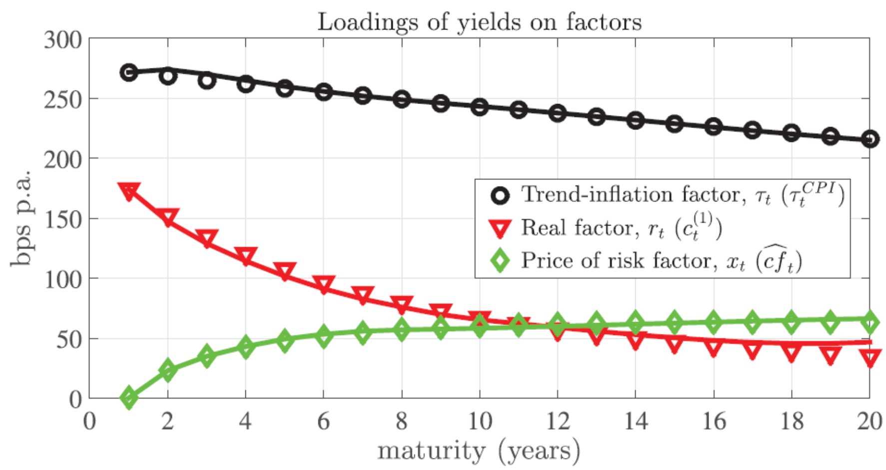


Figure 4
Loadings of yields on factors: regressions vs. affine model
The solid lines present the loadings of yields on observable factors $\tilde{F}_t = \left(\tau_t^{CPI}, c_t^{(1)}, \widehat{cf}_t\right)'$ obtained from Regression (30). The markers present the loadings obtained from the affine model given in Equation (17) for factors $F_t = (\tau_t, r_t, x_t)'$. The parameters of the affine model are calibrated by minimizing the sum of squared distances between the loadings from the regression and from the affine model, see Equations (31)–(32).

2.2.5. Liquidity in Treasury Markets

- Treasury markets are among the most liquid markets in the world.

- Nevertheless, there are interesting price differences between seemingly similar bonds, that is, bonds that are supposedly very close substitutes.

  This again points to downward-sloping demand curves, even in very liquid Treasury markets.

- Krisnamurthy (2002) studies the 30-year Treasury market. Here is the yield curve for 30-year bonds, issued just months apart

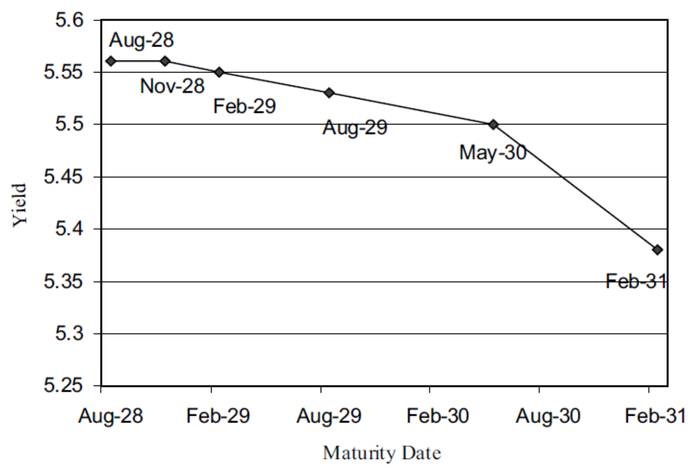


Fig. 1. The yield curve for the 30-year bond sector as of February 9, 2001.

- The yield spread between the new bond (Feb 31) and the previous new bond (May 30) is 12bp, while it is only 3bp if you go back one more vintage. Hence, the new bond (*on-the-run*) seems to be trading at a higher price, a "liquidity premium."

- The dynamics of the old bond- new bond spread between auction dates (vertical lines):

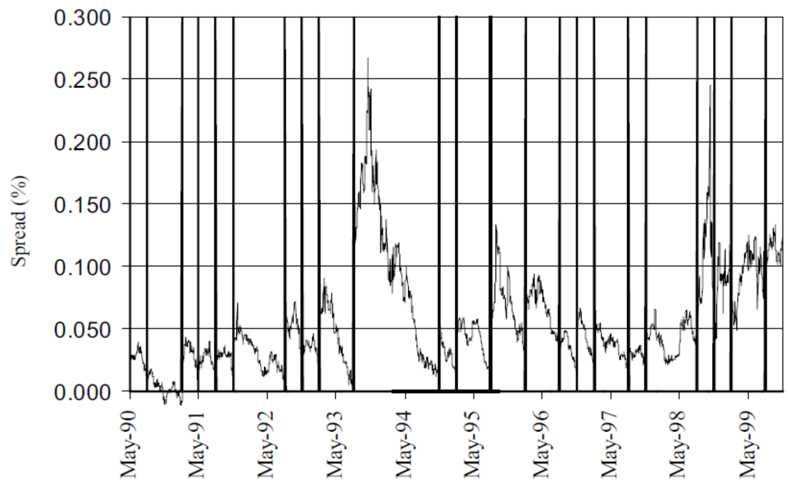


Fig. 2. Yield spread between bond and old-bond. The vertical lines mark auction dates.

- The spread widens right after the auction and narrows before the next auction.

- The convergence trade (buy the old bond, short the new bond) typically makes money. However, in some cases, like in the Fall of 1998, the spread widens, leading to losses. This is precisely when the hedge fund LTCM went under.

- Krishnamurty's conclusion: Convergence trade is not profitable on average due to the cost of shorting (repo rates). When the spread is the highest, repo rates are high as well, and shorting the new bond is expensive.

- What drives the joint dynamics of repo rates and the new-old bond spread? Krisnamurthy (2002) develops a simple model with segmented markets.

- The zero-coupon yield curve is extracted from coupon-bonds.
- The typical procedure is to estimate, on each day, a parametric model of the yield curve from the *cross-section* of bond prices. Example of a parametric model for the forward curve (Svensson, 1994):

$$f(n;\theta) = \beta_0 + \beta_1 \exp(-n/\tau_1) + \beta_2 (n/\tau_1) \exp(-n/\tau_1) + \beta_3 (n/\tau_2) \exp(-n/\tau_2),$$

  where $\theta = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ and n is the bond's maturity. A simpler model of this is Nelson and Siegel (1987).
- This procedure differs from standard affine models by:
  1. Not imposing no-arbitrage restrictions.
  2. Not modeling the dynamics of the factors over time. It is a purely cross-sectional model.
- To estimate the parameters θ, we can use non-linear least squares

$$\widehat{\theta}_t = \arg\min \sum_{i=1}^{N_t} \left[(P^i(\theta) - P^i_t) \frac{1}{D_i} \right]^2 .$$

  where P^i_t is the observed bond price, $P^i(\theta)$ the price implied by the model, D_i the duration of the bond, and N_t the number of bonds at time t.
- On normal days, this procedure fits very well. On crisis days, when liquidity dries up and arbitrage capital is limited, prices may deviate.

- Hu, Pan, Wang (2013) look at the dispersion in yields of individual bonds around a smooth yield curve as a measure of liquidity and arbitrage capital.

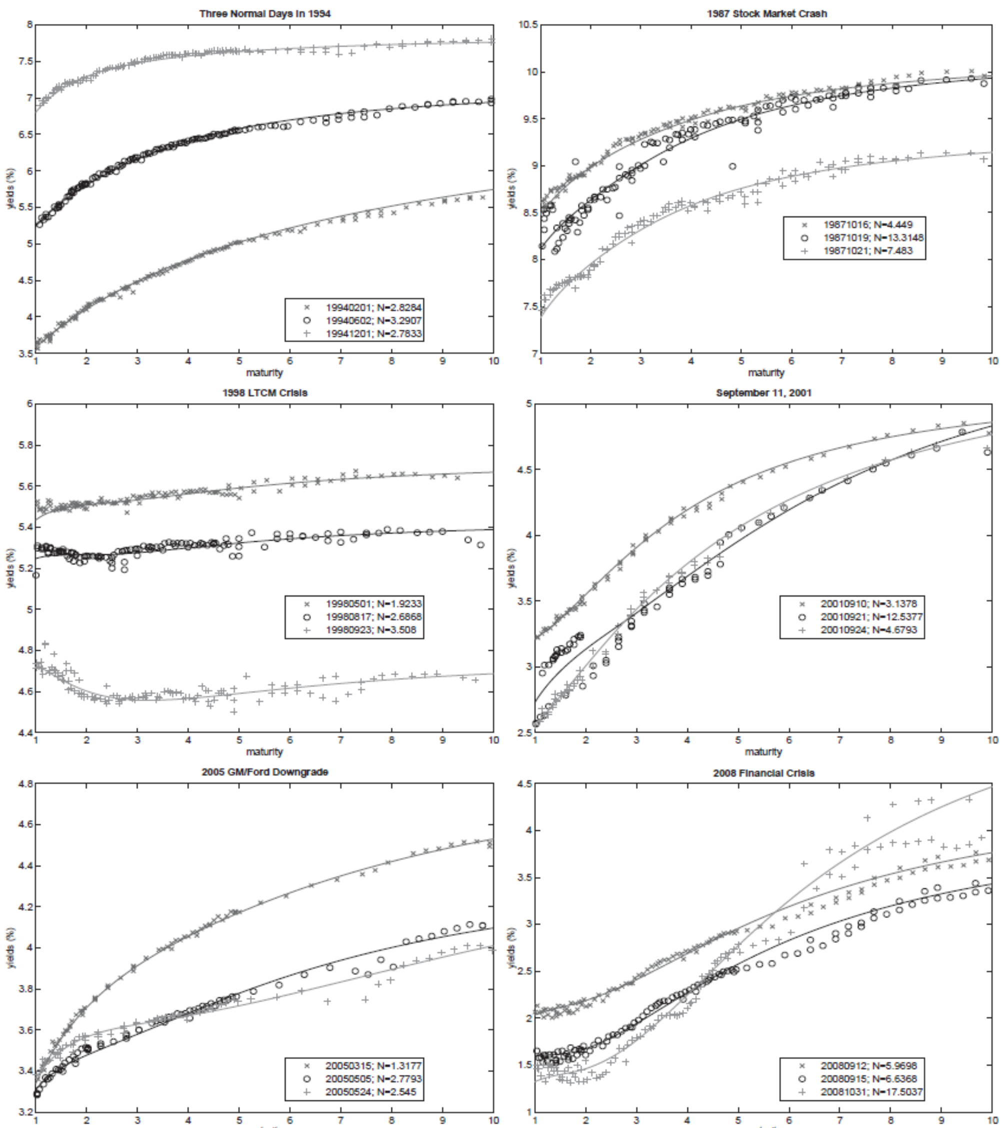


Figure 1. Examples of par-coupon yield curves and the market-observed bond yields, marked by "x", "o", or "+". The top left panel plots three random days in 1994, and the other five panels focus on the days surrounding five events: the 1987 stock market crash, the 1998 LTCM crisis, the September 11, 2001 terrorist attack, the 2005 GE/Ford downgrade, and the Lehman default in September 2008. Marked in the legends are the date of observation and the level of the noise measure for that day.

- The "Noise" measure is then constructed as

$$Noise_t = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}(y_t^i - y_i(\widehat{\theta}_t))^2}$$

- The dynamics of the noise measure over time

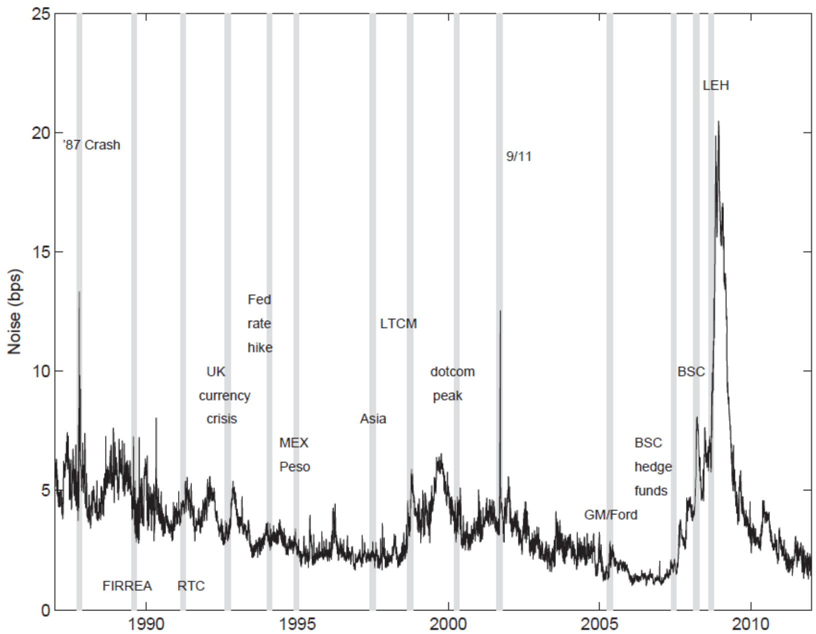


Figure 2. Daily time-series of the noise measure (in basis points). FIRREA: the Financial Institutions Reform, Recovery, and Enforcement Act of 1989; RTC: the Resolution Trust Corporation.

- The frictions are small most of the time. However, in times of crises, the noise measure spikes.

- Zooming in on the financial crisis

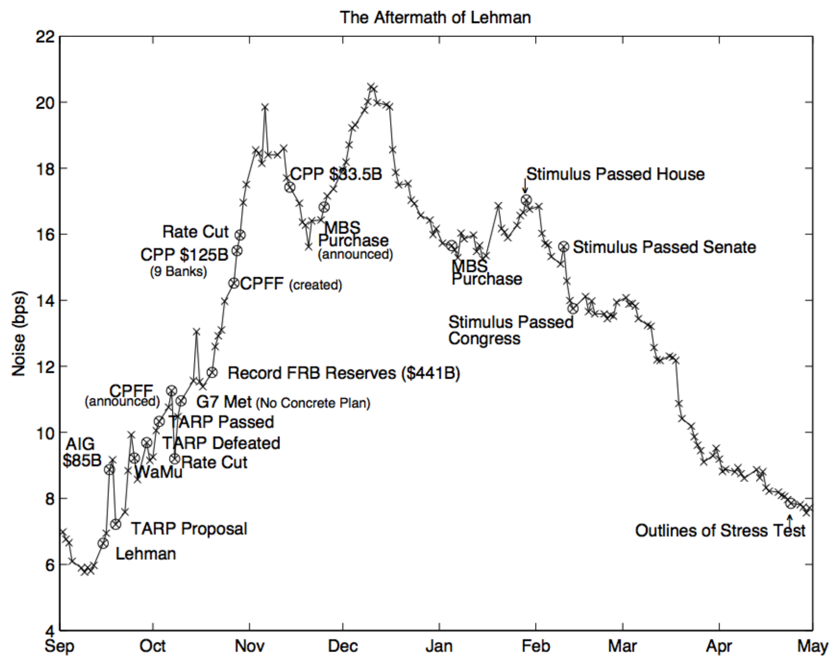


Figure 3. Daily time-series of the noise measure in late 2008 and early 2009. TARP: Troubled Asset Relief Program; CPP: Capital Purchase Program; CPFF: Commercial Paper Funding Facility; and the MBS Program is the Fed's $1.25 trillion program to purchase agency mortgage-backed securities.

- This bond illiquidity measure helps to explain the cross-section of hedge fund returns and the currency carry trade, both of which are sensitive to liquidity conditions.

- See Lou, Yan, and Zhang (2013) for more on frictions / downward-sloping demand curves in Treasury markets by assessing the price dynamics around Treasury auctions.

2.3. *Market Structure and Main Investors*

- From the flow of funds Table L.210.

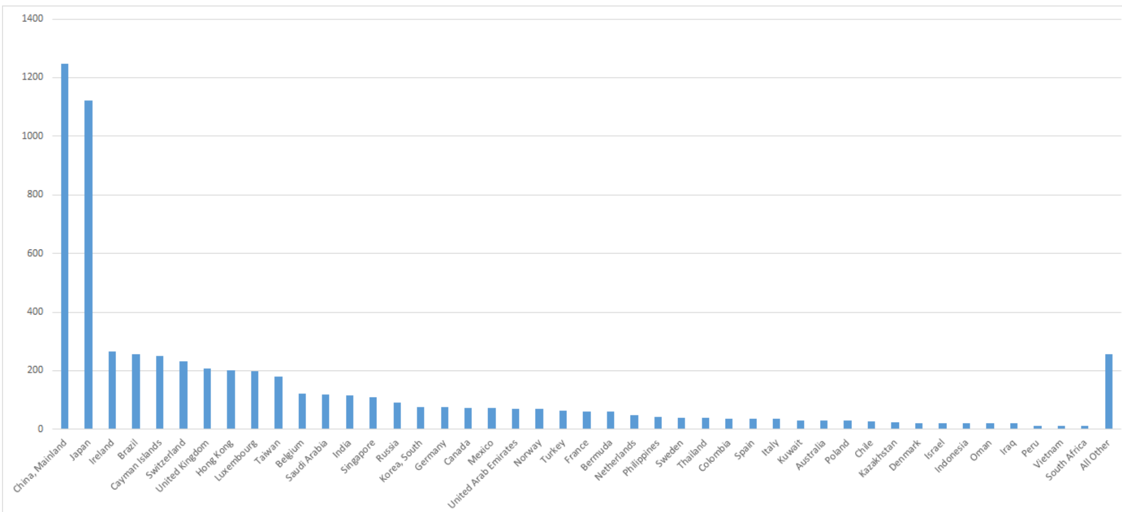
Name	Q2 2020	Q1 2020	Q2 2019
▼ Total liabilities	22,370,663	19,518,424	17,814,288
▶ Marketable Treasury securities	19,867,035	17,115,829	15,884,286
▶ Nonmarketable Treasury securities	2,503,628	2,402,595	1,930,002
▼ Total assets	23,514,344	20,699,167	18,058,419
▶ Household sector	1,814,091	1,792,252	2,056,418
Nonfinancial corporate business	88.379	69.026	50.110
Nonfinancial noncorporate business	86.487	84.253	76.536
▶ State and local governments	834,745	740,148	676,676
▶ Monetary authority	4,807,860	3,757,408	2,315,006
U.S.-chartered depository institutions	926,646	724,403	592,153
Foreign banking offices in U.S.	117,052	116,448	129,624
Banks in U.S.-affiliated areas	17,602	14,679	14,426
Credit unions	40,200	38,741	34,243
▶ Property-casualty insurance companies	178,056	168,583	156,954
▶ Life insurance companies	242,014	243,875	206,437
▶ Private pension funds	449,731	452,542	391,281
▶ Federal government retirement funds	2,157,483	2,156,627	1,713,134
State and local govt. retirement funds	337,620	352,106	388,832
▶ Money market funds	2,349,762	1,267,789	743,468
▶ Mutual funds	1,215,638	1,114,123	1,204,069
Closed-end funds	2,848	2,672	3,650
Exchange-traded funds	271,815	262,949	214,633
Government-sponsored enterprises	248,088	185,660	139,910
ABS issuers	30,568	32,025	26,530
Brokers and dealers	254,214	258,320	258,909
Holding companies	55,463	54,263	39,568
▶ Rest of the world	6,892,348	6,810,275	6,625,852
Discrepancy	-1,143,681	-1,180,743	-244,131

- Note the enormous expansion of Federal debt in 2020 (in large part due to the covid-19 crisis): +$4.5 trillion

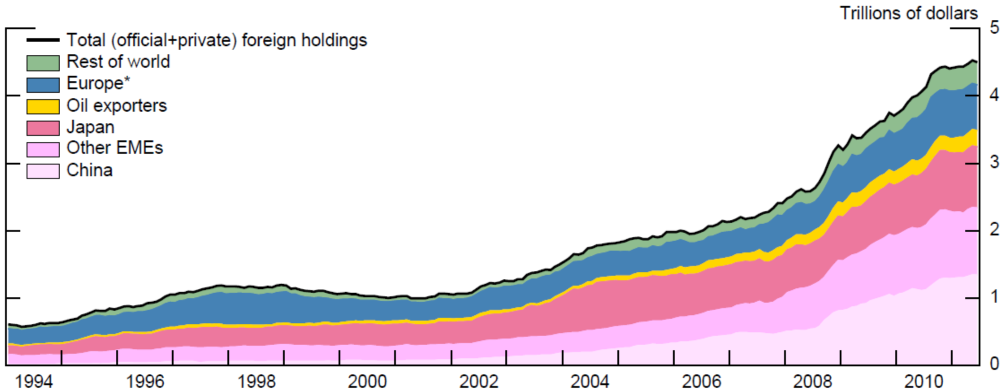
- The Fed's holdings increase by $2.5 trillion over the same period as its QE program expands in March 2020.

- Money Market Funds also expand by $1.6 trillion, tripling their size.

- Foreigners remain the largest owners of Treasuries at $6.8 trillion. Their share of Treasuries has been falling since 2008, from around 60% to 30%.
- The Treasury International Capital (TIC) System contains detailed data of global holdings of Treasuries (December, 2015 in billions of USD).



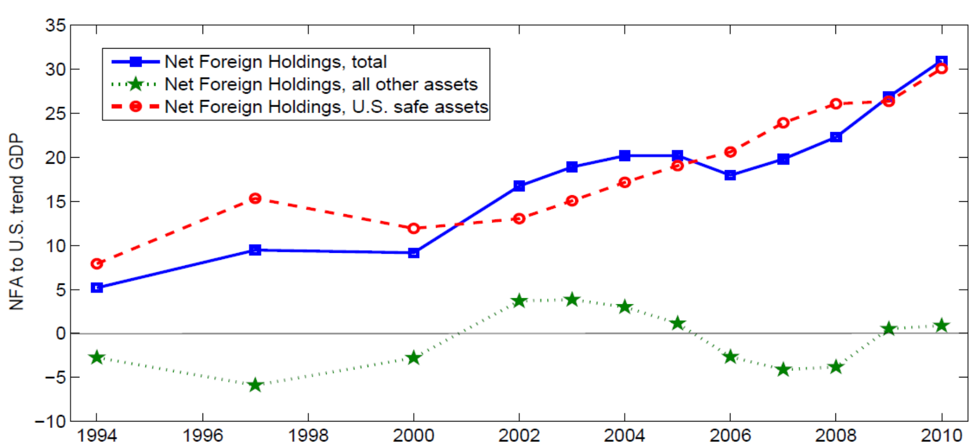
- Geography of foreign ownership (Beltran, Kretchmer, Marquez, and Thomas, 2012)



- The net foreign holdings trends are concentrated in safe assets; there are no such trends in other assets (Favilukis, Ludvigson, and Van Nieuwerburgh,2016)

Figure 3: Net Foreign Liabilities of the U.S. Relative to U.S. Trend GDP

The solid line (squares) denotes total net foreign holdings of long-term securities (the net foreign liability position of the U.S. in those securities) relative to U.S. trend GDP. Net foreign holdings are defined as foreign holdings of U.S. securities minus U.S. holdings of foreign securities. We define as safe the foreign holdings of U.S. Treasuries and Agencies. The dashed line (cicrles) denotes the thus constructed net foreign holdings in safe securities, while the dotted line (diamonds) denotes the net foreign holdings in all other securities. The data are from the U.S. Treasury International Capital System's annual survey of foreign portfolio holdings of U.S. securities. The data are available for December 1994, December 1997, March 2000, and annually from June 2002 until June 2010.



- There are more detailed sovereign bond holdings data available in the Euro area from Koijen, Koulischer, Nguyen, and Yogo (2017):

TABLE 1—SECURITY HOLDINGS BY COUNTRY GROUP AND INVESTOR SECTOR

Country group	Investor sector	Government bonds: Eligible	Government bonds: Ineligible	Corporate bonds: Investment	Corporate bonds: Speculative	ABS & covered bonds	Equity	Foreign	Total
Non-vulnerable	Insurance & pension	933	122	395	215	191	137	490	2,483
	Banks	815	325	535	154	702	127	681	3,339
	Mutual funds	577	175	296	250	189	900	2,422	4,809
	Households	19	12	98	150	12	465	148	904
	Other	125	76	36	47	26	767	90	1,167
	Total	2,469	709	1,360	817	1,121	2,396	3,830	12,702
Vulnerable	Insurance & pension	341	80	79	49	38	29	67	683
	Banks	508	343	190	233	588	72	292	2,226
	Mutual funds	161	120	48	50	25	156	809	1,369
	Households	174	61	123	241	5	199	75	878
	Other	113	41	12	25	2	257	39	489
	Total	1,296	647	452	598	658	713	1,281	5,645
Foreign		2,290	1,272	414	564	359	2,852		7,751
ECB		114	17	0	0	30	0	0	161

Note: Holdings reported in billion euros are time-series averages across five quarters from 2013Q4 to 2014Q4.

- Vulnerable countries are those that experienced a large increase in their CDS prices during the Euro crisis (e.g., Cyprus, Italy, Portugal, and Spain).

- Also in the Euro area, foreign investors play an important role, but their holdings are concentrated in the non-vulnerable countries.

2.4. *Interpreting the Facts*

2.4.1. Factor Models

- Factor models in bond markets come in the form of affine pricing models.
- In this case, we study the price level of bonds, while the factor models we have seen for equities focus on explaining difference in average returns (price changes).

  (Demand systems focus on the level of pricing, valuations).
- See Piazzesi (2010) for a review of affine pricing models.
- Each affine pricing model specifies 1.- 3. below. In case of the homoscedastic model

  1. Dynamics of the state variables:

     $$X_{t+1} = \Gamma X_t + \epsilon_{t+1},$$

     where $\epsilon_t \sim N(0, \Sigma)$.
  2. Link from the state variables to the short rate:

     $$y_t(1) = \delta_0 + \delta_1' X_t.$$

  3. Link from the state variables to the market prices of risk:

     $$\Lambda_t = \Lambda_0 + \Lambda_1 X_t.$$

  4. Model of the stochastic discount factor:

     $$\ln M_{t+1} = -y_t(1) - \frac{1}{2}\Lambda_t' \Sigma \Lambda_t - \Lambda_t' \epsilon_{t+1.}$$

5. Bond prices are computed as:

$$P_t(n) = \mathbb{E}_t \left[\prod_{s=1}^{n} M_{t+s} \right].$$

- This model implies

$$P_t(n) = \exp(A(n) + B(n)' X_t),$$

where $A(n)$ and $B(n)$ satisfy a set of recursions starting with $A(1) = -\delta_0$ and $B(1) = -\delta_1$.

- To compute the recursions, solve

$$P_t(n) = \mathbb{E}_t(M_{t+1} P_{t+1}(n-1)),$$

which expresses $A(n)$ and $B(n)$ in terms of $A(n-1)$ and $B(n-1)$.

- When $\Lambda_1 = 0$, risk premia are constant.
- The model can be extended with time-varying volatility, see Duffee (2002) (essentially affine models).
- The model can be estimated with maximum likelihood or GMM.
- Instead of using latent factors, there is also a literature using observable macro factors, starting with Ang and Piazzesi (2003).

- Recent topics in the affine term structure (ATS) literature:
  - Parameter identification: Joslin, Singleton, and Zhu (2011).
  - Unspanned macro-factors: Joslin, Priebsch, and Singleton (2014).
  - Maximum Sharpe ratio is too high: Duffee (2010).
  - Excess volatility of long-term yields: Giglio and Kelly (2017).
- These ATS models face potential challenges with the different regimes we have seen in bond markets
  1. Rising and falling inflation, peaking in the early eighties.
  2. Growing presence of foreign investors, which may affect the pricing of risks in the Treasury market (and hence Λ_t).
  3. Zero lower bound (ZLB) during 2008-2017 and again starting in March 2020.
  4. Shifts in the correlation between GDP growth and inflation from negative pre-2000 to positive post-2000 (Bilal 2017).

2.4.2. SDFs Based on "A" Marginal Investor

- The broker-dealer model of Adrien, Etula, and Muir (2014) explains the cross-section of Treasury returns as we have seen already.
- But broker-dealers only hold $250 billion of the total $22,370 billion in U.S. Treasuries outstanding, a mere 1%.
- The major players are:
  - The foreign sector (30.5%).
  - The monetary authority (21.5%)
  - MMMF + regular mutual funds (15.9%).
  - Households (8.1%).
  - Long-term investors (insurance companies, pension funds).
- A key question is to understand what drives the demand from the long-term investors and the foreign sector.
- See Favilukis, Ludvigson, and Van Nieuwerburgh (2016) for a model of foreign purchases of U.S. Treasuries and their equilibrium implications for U.S. investors and asset prices.
- See Elenev, Landvoigt, Schultz, and Van Nieuwerburgh (2021) for an equilibrium model of quantitative easing and the term structure, with a central role for intermediaries whose SLR and LCR constraints are affected when QE expands their reserves.
- The empirical work in this area is much more limited.

- Example of a simple model of long-term investors: Domanski, Shin, and Sushko (2017).
- Consider an insurer or pension fund with payments C that grow at a rate g over time.
- The present value of liabilities is:

$$L = \sum_{s=1}^{\infty} C \frac{(1+g)^s}{(1+r)^s} = \frac{C}{r-g}.$$

- Balance sheet identity: $M + B = L + E$, where M is cash, B are bonds, and E is equity.
- The value of bonds is equal to $B = qP$, where $P = (1+r)^{-T}$ and q the number of bonds.
- Assume that the insurer is subject to a risk constraint, and has to match the duration of assets and liabilities perfectly.
- (One can allow for some limited duration mismatch and assume that the investor acts subject to a risk constraint).
- Duration of the liabilities is:

$$-\frac{\partial L}{\partial r}\frac{1}{L} = \frac{1}{r-g}.$$

- Duration of the bond portfolio with maturity T is:

$$-\frac{\partial P}{\partial r}\frac{1}{P} = \frac{T}{1+r}.$$

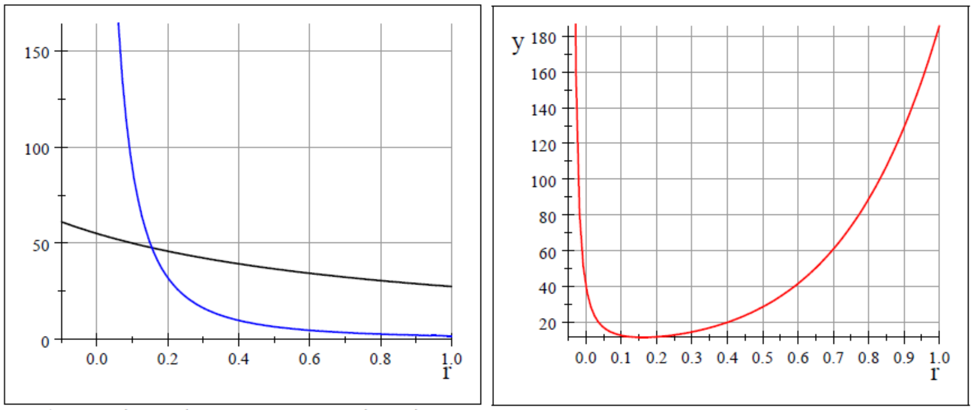
- Perfect duration hedging (immunization) imposes the restriction:

$$Pq\frac{T}{1+r} = L\frac{1}{r-g}$$

- This pins down the demand for bonds q

$$q(r) = \frac{C(1+r)^{T+1}}{T(r-g)^2}.$$

- Demand:



Assets (black) and liabilities (blue). Holding of the benchmark bond y.

Figure 3: Convexity of assets and liabilities, keeping holdings of benchmark bond fixed (left) and holding of benchmark bond in the immunising portflio (right); for T=10, C=0.5, and g=-0.05.

- The convexity of the liabilities is much higher; property of cash flows that are spread out compared to a single payment like with a zero-coupon bond.

- The striking insight is that as interest rates fall, and hence bond prices rise, the demand of long-term investors may in fact *rise*!

- Intuition: they need to buy bonds to immunize the portfolio against further interest rate increases.

- This means that demand curves can be upward-sloping.

- Domanski, Shin, and Sushko (2017) use detailed holdings data from German insurance sector to provide some evidence consistent with the model: upward-sloping demand in 2013-14

Figure 10: Demand elasticity (duration weighted), long-term government bond holdings of German insurance sector; OECD government bonds, $<$10 year durations.

- Insurers adjust their duration more than other investors

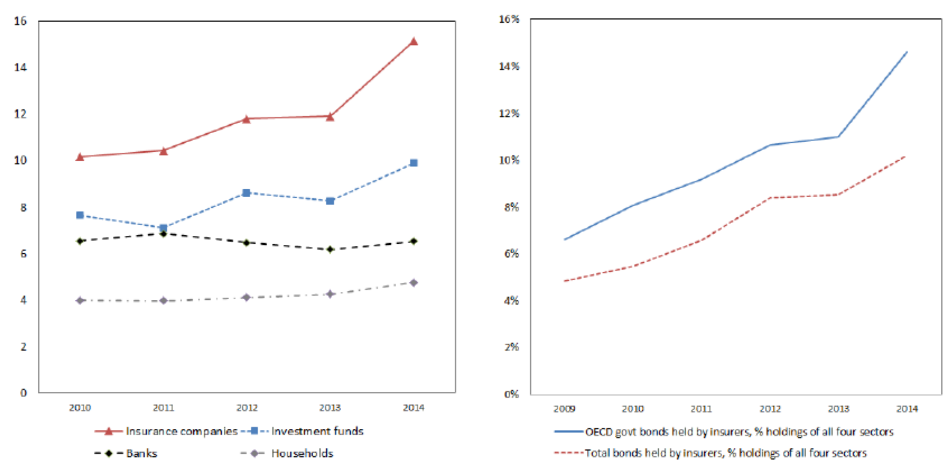


Figure 7: Comparison of bond portoflio duration between insurance companies and other major investor sectors; and trends in OECD government bond holdings relative to other major investor sectors. Based on bond portfolio allocation data of German insurance companies, investment funds, banks, and households

- Ozdagli and Wang (2020) propose a similar model in which life insurance companies do duration matching under adjustment costs. The model predicts that insurers tilt their portfolios towards higher yielding corporate bonds when rates decline. They do so because higher-yielding bonds have longer duration, and this portfolio tilt closes the duration gap. In the process they take on more credit risk.

2.4.3. Consumption- and production-based equilibrium models

- For each class of consumption-based AP models, there is a term structure paper:
  - Habits: Wachter (2006) and Le, Singleton, and Dai (2010).
  - Long-run risks: Piazzesi and Schneider (2006), Bansal and Shaliastovich (2013) and Kung (2015).
  - Disasters: Gabaix (2012).
  - Production model with recursive preferences (estimated): Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2012).
- These models, with varying success, match salient features of the yield curve related to the slope of the yield curve and bond excess return predictability.
- Real term structure tends to be downward sloping since real bonds are a hedge (real rates rise in good times, real bond prices fall). Generating an upward-sloping nominal term structure requires inflation to be high in bad times: $Cov(\Delta c, \pi) < 0$.

2.4.4. Demand-based models

- Vayanos and Vila (2021) propose a model where one group has an exogenous demand for bonds of a given maturity with an inelastic component to it.
- Question: Who are the inelastic investors (pension funds, foreign investors?) and who are the elastic investors (mutual funds, households?)?
- An arbitrageur smoothes out arbitrage opportunities along the yield curve.
- The yield curve reflects the demand shocks of inelastic investors as well as the (exogenous) short rate.
- Through artfully chosen preferences and demand shocks, the model results in an affine term structure model.
- Outline of the model
  - There is an exogenous short-term (instantaneous) interest rate (perfectly elastic supply of short-term bonds), r_t, which is an AR(1) in continuous time

$$dr_t = \kappa_r(\overline{r} - r_t)dt + \sigma_r dZ_{rt}.$$

  - In addition, there is a continuum of bonds with maturities in $[0, N]$. The bonds are in zero net supply.
  - One group of investors have dollar demand, $Q_t(n)$, for bonds of a given maturity at a given point in time

$$Q_t(n) = \alpha(n)n(y_t(n) - \beta_t(n)),$$

where $\alpha(n)$ is positive (downward-sloping demand) and constant over time, but may vary across maturity.

– Two observations about the demand curves

  1. The second part of the demand, $\beta_t(n)$, is inelastic and also varies over time. This can be motivated by hedging demands.

     This is a modern version of "preferred habitat" models of the term structure. E.g., insurers like very long-term bonds for their duration-matching benefits.

  2. The demand for bond n only depends on the price of bond n, not on the bonds of other maturities.

– Structure of the demand shocks

$$\beta_t(n) = \overline{\beta} + \theta(n)\beta_t,$$

where the demand factor, β_t, is also an AR(1)

$$d\beta_t = -\kappa_\beta \beta_t dt + \sigma_\beta dZ_{\beta t}.$$

Note: We could allow for multiple factors here.

– We have introduced two yield factors: r_t and β_t.

– To close the model, we introduce a group of arbitrageurs.

– The arbitrageurs are myopic, unconstrained investors with mean-variance preferences.

– Arbitrageurs are the only agents who can invest in bonds of all maturities. But their risk aversion induces **limits to arbitrage**.

– The arbitrageurs' wealth evolves as

$$dW_t = \left(W_t - \int_0^N Q_t^A(n)dn \right) r_t dt + \int_0^N Q_t^A(n) \frac{dP_t(n)}{P_t(n)} dn,$$

where $Q_t^A(n)$ is the dollar demand of the arbitrageurs for a bond with maturity n.

– Arbitrageurs choose their portfolio to maximize

$$\max_{(Q_t^A(n))_{n\in[0,N]}} E_t(dW_t) - \frac{a}{2} Var_t(dW_t),$$

where a is the risk aversion coefficient.

– Note that we are modeling dollar demand throughout, which is what we usually do in M-V/CARA models.

– Market clearing implies:

$$Q_t(n) = Q_t^A(n),$$

since bonds are in zero net supply.

The left-hand side of this equation is exogenous.

- To solve the model:

1. Conjecture that bond prices are exponentially affine

$$P_t(n) = \exp(-A(n) - A_r(n)r_t - A_\beta(n)\beta_t).$$

2. Derive the arbitrageurs' first-order condition. The presence of unconstrained arbitrageurs ensures the absence of arbitrage opportunities.

3. Use the market clearing condition to solve for the unknown parameters.

- Useful insights
  - When the preferred habitat investors demand a lot of a certain bond, this has an impact on the entire term structure. There are only two factors and hence bonds are close substitutes. Arbitrageurs care about the total duration and demand risk in the market.
  - If risk aversion $a \simeq 0$, short-rate shocks dominate and demand shocks are relatively unimportant
    $\Rightarrow$ There is an approximate one-factor structure; all that matters is the total amount of duration risk in the market.
  - Higher levels of risk aversion make arbitrageurs less willing to substitute across maturities and can lead to more local effects ("habitat" effects), especially in the presence of multiple demand factors.

- Instead of demand shocks, one can also think of shocks to the **supply** of bonds outstanding across maturities.
- In Greenwood and Vayanos (2014), they replace the exogenous demand by net supply coming from the government and the other investors:

$$S_t(n) = \xi(n) + \theta(n)\beta_t$$

- Market clearing:

$$S_t(n) = Q_t^A(n).$$

- This is important in the context of QE because the FED/ECB is changing the residual supply of bonds via asset purchase programs.
- In principle, one can disaggregate $S_t(n)$ into different investors and estimate a demand system. This would tell us the importance of different investor groups for the demand in Treasury markets.
- Main predictions
  1. Yields increase with the dollar duration of bond supply, controlling for the short rate.
  2. Bond risk premia increase with the dollar duration of bond supply.

- Greenwood and Vayanos (2014) measure the dollar duration of the outstanding supply at any given point in time by the maturity-weighted government debt portfolio duration

$$\frac{MWD_t}{GDP_t} = \frac{\sum_{0 \leq n \leq 30} D_t(n)n}{GDP_t},$$

where $D_t(n)$ are the dollar payments of all U.S. government debt,

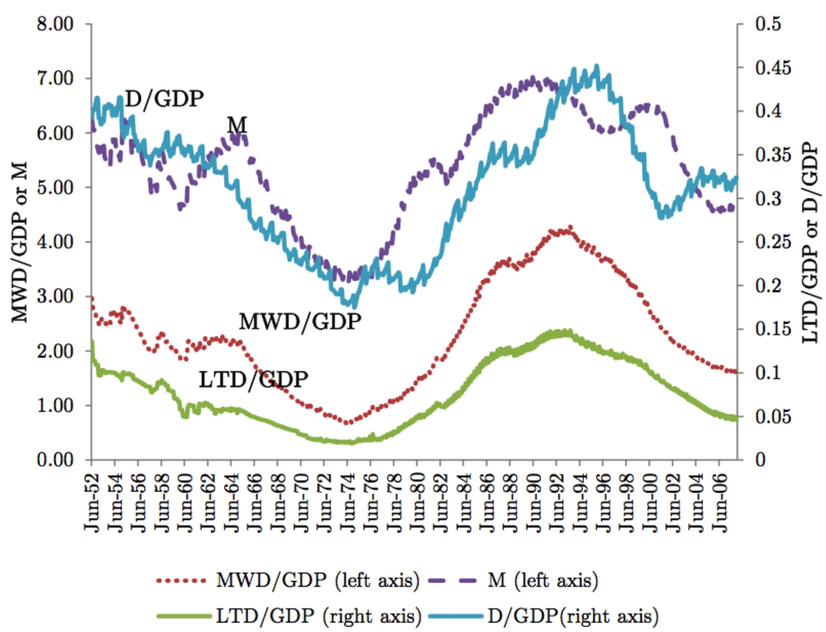
$$D_t(n) = PR_t(n) + C_t(n),$$

where $PR_t(n)$ is the total principal payment in n periods and $C_t(n)$ the total coupon in n years.

- The dynamics of bond supply

Figure 2
Bond supply, 1952-2007

MWD/GDP is the maturity-weighted-debt-to-GDP ratio, computed by multiplying each debt payment by the corresponding maturity, summing across maturities, and scaling by GDP. LTD/GDP is the long-term-debt-to-GDP ratio, computed by summing all debt payments with maturity beyond ten years, and scaling by GDP. M is the dollar-weighted average maturity expressed in years. D/GDP is the ratio of the aggregate principal payments of all Treasury securities to GDP. MWD/GDP, LTD/GDP, and M are computed using aggregate principal and coupon payments.



- To test the predictions on yields, consider the following regression

$$y_t(n) = a + b\frac{MWD_t}{GDP_t} + cy_t(1) + u_t.$$

Table 2
Bond supply, bond yields and bond returns

Monthly time-series regressions of the form:

$$y_t^{(\tau)} = a + bX_t + cy_t^{(1)} + u_t$$

$$r_{t+k,k}^{(\tau)} = a + bX_t + cy_t^{(1)} + u_{t+k}$$

The dependent variable is the yield or the one-year, three-year, or five-year return of the τ-year bond. The independent variable X_t is *MWD/GDP*, the maturity-weighted-debt-to-GDP ratio, or *LTD/GDP*, the long-term-debt-to-GDP ratio. The regressions control for the one-year yield. The first set of t-statistics, reported in brackets, are based on Newey-West standard errors with 36 lags in the case of the yield and one-year return regressions, and 54 and 90 lags in the case of the three- and five-year return regressions. The second set of t-statistics are based on modeling the error process as AR(1) for the yield regressions, and as ARMA(1,k) for the return regressions where k denotes the number of months in the return cumulation (e.g., twelve for the one-year return).

	X=*MWD/GDP*							X= *LTD/GDP*						
	b	[t NW]	[t AR]	*c*	[t NW]	[t AR]	R^2	*b*	[t NW]	[t AR]	*c*	[t NW]	[t AR]	R^2
Yield spreads:														
Yield 2-yr bond	0.001	[2.597]	[2.363]	0.981	[50.113]	[70.970]	0.987	0.029	[2.476]	[2.293]	0.982	[49.381]	[69.955]	0.987
Yield 3-yr bond	0.002	[2.510]	[1.881]	0.951	[29.510]	[36.999]	0.968	0.044	[2.364]	[1.811]	0.952	[29.150]	[36.591]	0.968
Yield 4-yr bond	0.002	[2.497]	[1.805]	0.932	[22.657]	[30.645]	0.949	0.058	[2.356]	[1.772]	0.934	[22.419]	[30.362]	0.948
Yield 5-yr bond	0.002	[2.358]	[1.580]	0.913	[19.528]	[23.621]	0.933	0.064	[2.258]	[1.601]	0.914	[19.387]	[23.506]	0.932
Yield LT bond	0.004	[2.682]	[1.719]	0.795	[12.167]	[12.993]	0.379	0.107	[2.610]	[1.822]	0.797	[12.234]	[13.253]	0.374

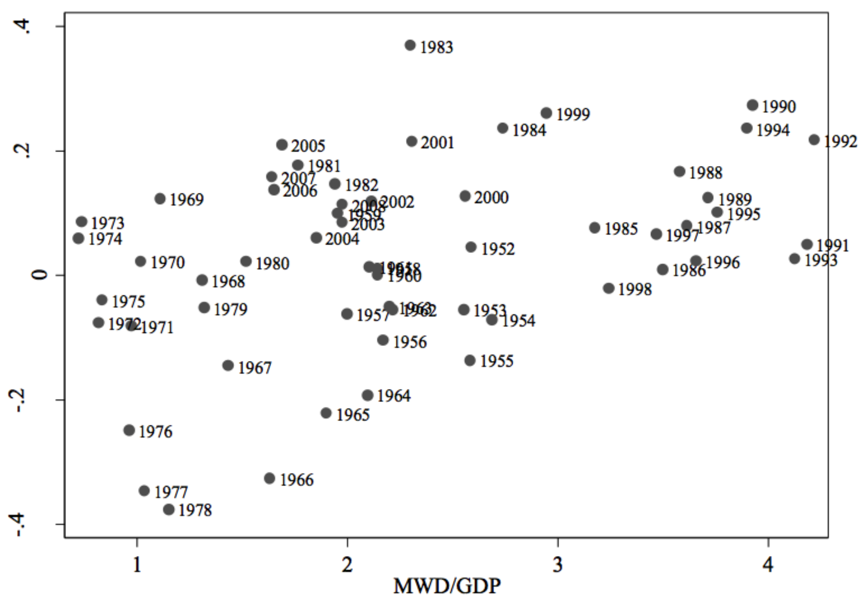
- LTD/GDP only accounts for long-term debt.

- Bond supply and excess return on long-term bonds over the subsequent 3 years

Figure 4
Bond supply and excess bond returns

Plots of three-year holding-period excess return on long-term government bonds (vertical axis) against the maturity-weighted-debt-to-GDP ratio (horizontal axis). Panel A shows the 1952-2007 period. Panel B shows the 1916-1941 period.

Panel A. 1952-2007



- We can turn this into regressions of the form

$$rx_{t,t+k}(n) = a + b\frac{MWD_t}{GDP_t} + cy_t(1) + u_{t+k}.$$

Table 2
Bond supply, bond yields and bond returns

Monthly time-series regressions of the form:

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	X=*MWD/GDP*							X= *LTD/GDP*						
	b	[t NW]	[t AR]	*c*	[t NW]	[t AR]	R^2	*b*	[t NW]	[t AR]	*c*	[t NW]	[t AR]	R^2
Yield spreads:														
Yield 2-yr bond	0.001	[2.597]	[2.363]	0.981	[50.113]	[70.970]	0.987	0.029	[2.476]	[2.293]	0.982	[49.381]	[69.955]	0.987
Yield 3-yr bond	0.002	[2.510]	[1.881]	0.951	[29.510]	[36.999]	0.968	0.044	[2.364]	[1.811]	0.952	[29.150]	[36.591]	0.968
Yield 4-yr bond	0.002	[2.497]	[1.805]	0.932	[22.657]	[30.645]	0.949	0.058	[2.356]	[1.772]	0.934	[22.419]	[30.362]	0.948
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Returns:														
1-year return 2-yr	0.004	[1.979]	[1.438]	1.114	[12.201]	[11.290]	0.774	0.116	[2.176]	[1.590]	1.118	[12.214]	[11.245]	0.776
1-year return 3-yr	0.007	[1.860]	[1.512]	1.134	[6.751]	[6.847]	0.507	0.191	[2.013]	[1.627]	1.140	[6.750]	[6.804]	0.509
1-year return 4-yr	0.010	[1.964]	[1.774]	1.157	[4.864]	[4.535]	0.358	0.266	[2.084]	[1.867]	1.166	[4.855]	[4.495]	0.360
1-year return 5-yr	0.011	[1.902]	[1.852]	1.145	[3.897]	[4.172]	0.263	0.308	[2.012]	[1.913]	1.154	[3.897]	[4.132]	0.265
1-year return LT bond	0.026	[3.097]	[3.462]	1.212	[2.846]	[3.181]	0.190	0.685	[3.196]	[3.468]	1.229	[2.860]	[3.142]	0.189
3-year return LT bond	0.065	[4.200]	[4.121]	3.737	[4.971]	[4.587]	0.506	1.786	[4.200]	[4.284]	3.795	[5.039]	[4.627]	0.516
5-year return LT bond	0.094	[5.421]	[3.580]	6.139	[5.401]	[4.650]	0.658	2.625	[5.340]	[4.068]	6.235	[5.612]	[5.062]	0.675

- Greenwood, Hanson, and Vayanos (2015) extend this model to think about forward guidance, which is modeled as information (signals) about future short rates and supply.

2.4.5. Bond Yields and the Macro-Economy

- In most models, bond yields embed information about future growth. For instance, think of the standard consumption-CAPM

$$y_t(1) = -\ln\beta + \gamma\mathbb{E}_t[\Delta c_{t+1}] - \frac{1}{2}\sigma_c^2\gamma^2.$$

- A large literature looks at the link between the term structure of interest rates and future growth.

- From Koijen, Lustig, and Van Nieuwerburgh (2017):

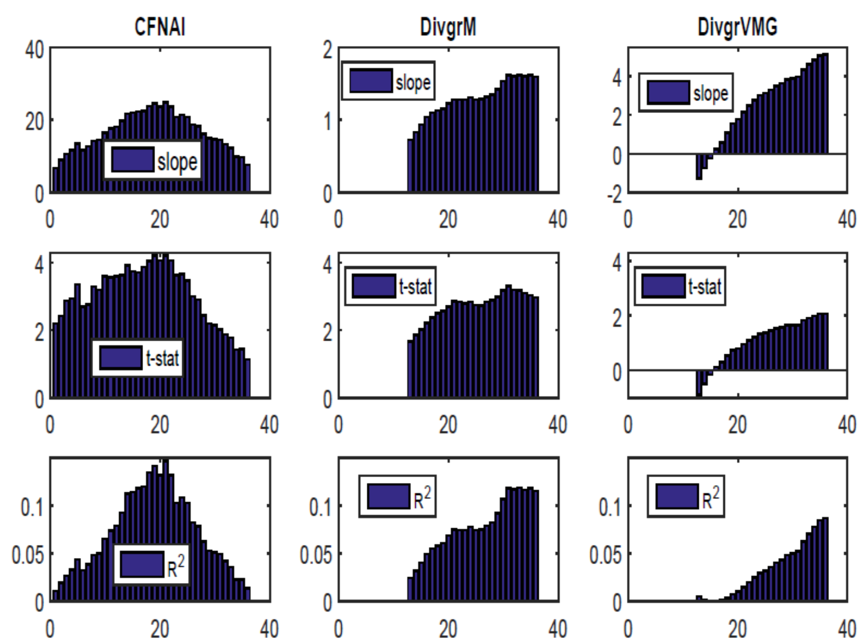


Figure 3: Economic activity predicted by bond factors.

We consider a regression of future values of $CFNAI$, which we normalize to have mean zero and standard deviation one, on the current CP factor:

$$CFNAI_{t+k} = c_k + \beta_k CP_t + \varepsilon_{t+k}, \quad (2)$$

where k is the forecast horizon expressed in months. The regressions are estimated by OLS and we calculate Newey-West standard errors with $k-1$ lags. The top panel displays the predictive coefficient β_k, the middle panel the t-statistic, and the bottom panel the corresponding R^2. We consider $k = 1, \ldots, 36$ months of lags, displayed on the horizontal axis in each panel, and the t-statistics are computed using Newey-West standard errors with $k-1$ lags. In all three columns, the predictor is the CP factor. In the left column, $CFNAI_{t+k}$ is the dependent variable. In the middle column, the aggregate dividend growth rate Δd_{t+k} is the dependent variable. In the last column, the dividend growth rate on value minus growth $\Delta d^V_{t+k} - \Delta d^G_{t+k}$ is the dependent variable. The sample is March 1967 until December 2012.

3. Bond Prices and Unconventional Monetary Policy

- As interest rates hit zero, central banks resorted to unconventional monetary policy including forward guidance and, in particular, quantitative easing.

- Two useful papers to read as an introduction:

  – Woodford (2012).

  – Krisnamurthy and Vissing-Jorgensen (2011).

- Fast-growing literature with many open questions.

- Outline:

  1. Description various QE programs.
  2. Broad outline of theories.
  3. Evidence based on policy announcements.
  4. Evidence based on holdings and low-frequency diff-in-diff estimates.

- In the discussion, it is sometimes useful to distinguish:

  – Pure quantitative easing: Central bank purchases short-term bonds with newly-created reserves.

  – Operation twist: Central bank purchases long-term bonds and sells short-term bonds.

- At the zero-lower bound, reserves and short-term bonds are (almost) perfect substitutes. The distinction between these two programs vanishes.

- The objectives of central banks differ across countries:
  - Europe: Inflation (close to, but below, 2%).
  - U.S.: Maximum employment, stable prices, and moderate long-term interest rates.
  - Japan: Price stability and the stability of the financial system.
  - See here for a simple overview of the differences between the various central banks.
- Summary of QE programmes:
  - Federal Reserve in the U.S. (overview):
    * **QE1** (November 2008-March 2010, including the extension):
      - $100 billion of agency debt and $550 billion of mortgage-backed securities, the programme was subsequently expanded in March 2009 with $100 billion agency debt, $750 billion agency MBS, and $300 billion long-term Treasuries.
    * **QE2** (November 2010-June 2011): Buy $600 billion in long-term Treasuries.
    * **Operation twist** (September 2011-June 2012): Buy $400 billion in Treasuries with maturities between 72 and 360 months and sell an equal amount of Treasuries with maturities in the 3 to 36 months range.
    * **QE3** (September 2012-October 2014): Buy $40 billion per month in MBS.

- Evolution of the balance sheet of the FED:

Figure 2: Federal Reserve System Assets
January 2007 - December 2014

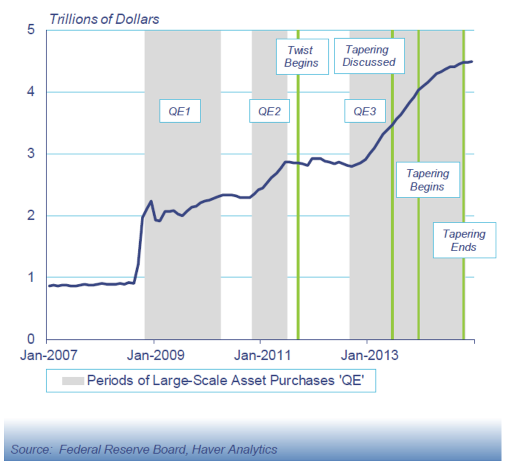
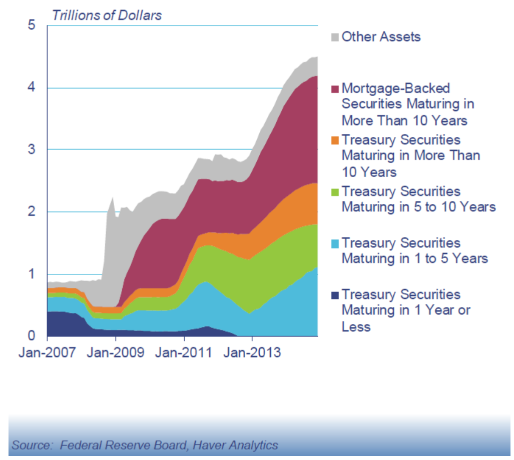


Figure 3: Federal Reserve System Assets
January 2007 - December 2014

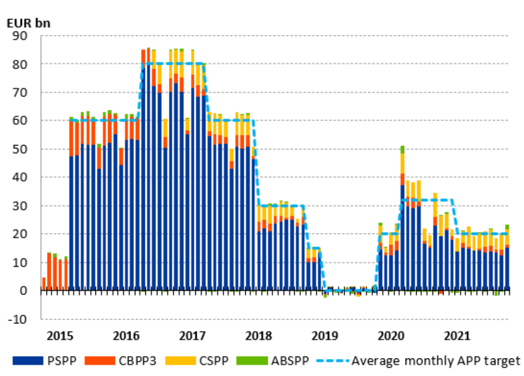


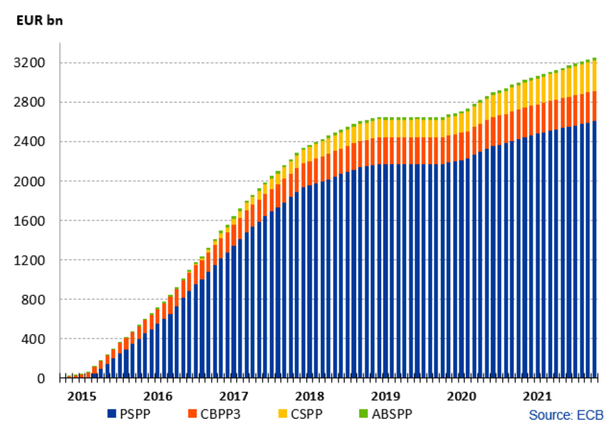
- A new QE program was launched in response to the covid-19 crisis on March 15, 2020 of at least $700 billion. Expanded on March 23, 2020.

- Fed also launches many other program including programs to finance Treasury's purchases of corporate loans and bonds (PMCCF, SMCCF, TALF) and the Paycheck Protection Program (PPP) making bridge loans to SMEs.

- Several new papers on effectiveness of PPP program. See Elenev, Landvoigt, and Van Nieuwerburgh (2021) for details on these programs and a GE model on how these programs helped the broader economy.

- ECB in the euro area:
  - Various asset purchase programme programmes, see here for an overview.
  - Main recent announcement in January 2015 is to purchase €60 billion per month until September 2016. Composition: €44 billion in government bonds, €6 in supranationals, €10 billion in covered bonds.
  - The programme has been extended multiple times and monthly purchases were scaled up to €80 billion per month. The programme is still ongoing and the programme has now been extended to include corporate bonds.
  - The purchases are financed through an increase in reserves.
  - Bond purchases halted in December 2018, but reinvestment of proceeds. Program restarted on Nov 1, 2019 at €280 billion per month.
  - Asset purchase programs expanded in response to covid crisis. PSPP holds €2.45 trillion as of November 5, 2021.





3.1. Schematic Overview of the Literature

- Three broad categories of theories: QE has no effect, QE can have a positive effect, and QE has a negative effect.
- In all cases, we briefly discuss the implications for asset prices and portfolio holdings.

1. <u>QE has no impact on prices and quantities</u>

   - These theories are closely connected to the Modigliani and Miller irrelevance theorems in corporate finance.
   - Two important papers: Wallace (1981) and Eggertsson and Woodford (2003).
   - The main insight is that when the central bank passes on all losses on its portfolio in the form of lump-sum taxes, then the portfolio of the central bank does not matter if households are unconstrained.
   - Households simply unwind the portfolio of the central bank.
   - The result requires frictionless trading, lump-sum taxes, no portfolio constraints, . . .
   - In the presence of heterogeneity, and in particular in the presence of global investors, it matters who is plausibly exposed to losses of the central bank.
   - **Predictions:** Central-bank purchases should be accommodated by investors that can be taxed. Consumption, the price level, all bond prices, and exchange rates are unaffected.

     Local investors rebalance.

2. QE has a positive impact on prices, price level, and real activity

- QE can have a positive impact in at least three ways:
  (a) Signalling/commitment
    - Forward guidance can be valuable at the zero lower bound. By promising to keep interest rates low for longer than necessary, investors increase consumption today, which increases demand and prices.
    - See Werning (2011) for a clean model of this idea. Note: His model is entirely deterministic, so the commitment problem is unrelated to uncertainty.
    - Buying long-term bonds may be interpreted as a commitment device. If the central bank rapidly raises interest rates, it would experience large (mark-to-market) losses on its portfolio. It is not obvious that mark-to-market losses are relevant for central banks. What really matters are defaults (e.g. on MBS and corporate bonds).
    - If all that QE does is to act like a commitment device, this may be a quite costly tool (Woodford, 2012).

(b) QE reduces the amount of duration risk in markets.

- By reducing the amount of duration risk, the term premium declines, and investors may be inclined to substitute to other (closely-related) securities like corporate bonds and mortgage-backed securities.
- This in turn lowers risk premia (e.g., the credit risk premium and the prepayment risk premium) in other markets and therefore lowers the borrowing costs for firms and households (in the mortgage market).
- This is often referred to as the portfolio rebalancing channel.
- The effects on the assets purchases, and the assets to which investors substitutes, depend on demand (cross-)elasticities.
- References as discussed before: Greenwood and Vayanos (2014) and Greenwood, Hanson, and Vayanos (2015).
- Note that these are not full general-equilibrium models as the interest rate is modeled exogenously and the demand by central banks "removes risks from markets," yet we are not modeling the budget constraint of the government.

(c) Brunnermeier and Sannikov (2016) propose a production economy with financial frictions and market segmentation.
   - Assuming that QE purchases can raise prices, it matters who holds these assets.
   - QE can have a positive impact on the price level and on growth by relaxing the constraints of financial intermediaries.
   - E.g., if QE in the Euro area raises bond prices that are held by banks in vulnerable countries (e.g. Portugal), it may strengthen the balance sheets of the intermediaries ("stealth recapitalization").

- Predictions:
  - Signalling has direct implications for prices, and should take place around the (surprise) policy announcement. Empirical challenge: Measuring surprises.
  - Portfolio rebalancing channel has implications for prices and portfolio holdings. To identify substitution effects, portfolio holdings are helpful as it can be useful to identify substitution patterns. E.g., who sells to the ECB and what do investors buy instead?
  - In the constrained intermediary story, in addition to identifying price effects, it matters who holds the securities purchased by the central bank. In the context of euro-area policy, this would mean that banks in vulnerable countries hold a lot of domestic government debt.

3. <u>QE may have negative effects</u>

- QE can have a negative impact in at least two ways:
  - (a) Reduction in the supply of safe assets.
    - Krishnamurthy and Vissing-Jorgensen (2012) provide evidence that there is a group of investors demanding "safe assets."
    - The spread between Treasuries and AAA securities, which are seemingly close substitutes, depends on the supply of Treasuries available. There seems to be a special demand for Treasuries.

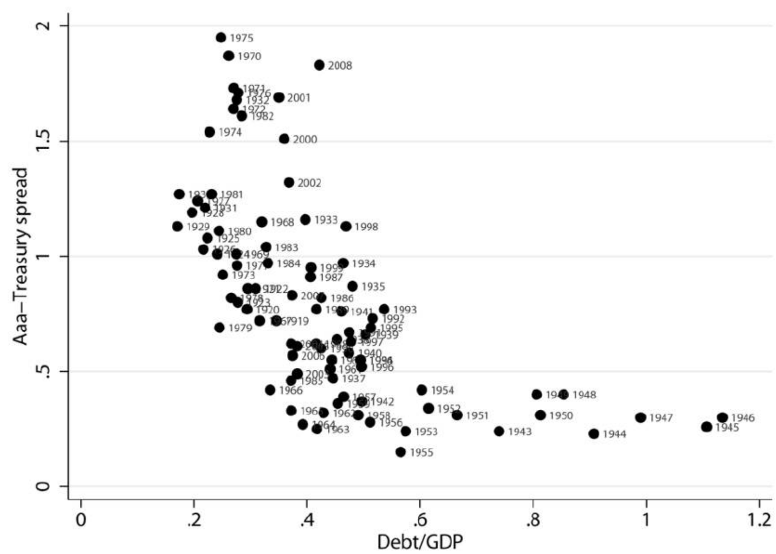


FIG. 1.—Corporate bond spread and government debt. The figure plots the Aaa-Treasury corporate bond spread (y axis) against the debt-to-GDP ratio (x axis) on the basis of annual observations from 1919 to 2008. The corporate bond spread is the difference between the percentage yield on Moody's Aaa long-maturity bond index and the percentage yield on long-maturity Treasury bonds.

    - If the FED buys lots of safe assets, this may actually decrease welfare as there is demand for safety, see also Krisnamurthy and Vissing-Jorgensen (2012).

(b) Risk shifting / reaching for yield.

  - When interest rates are low and yields are compressed, investors may substitute to other, riskier asset classes.

    ⇒ *In fact, this is the idea behind the portfolio-balance channel!*

  - However, some (regulated) financial institutions may start to take on too much risk, or risks may get too concentrated.
  - Ideally, this is addressed through sound capital and risk regulation. However, the regulation of financial institutions is often slow to adjust.
  - Main references: Woodford (2011), Stein (2014), and Coimbra and Rey (2017).
  - In this case, the **predictions** are mostly in the context of risk distribution and *risk concentration*.

- We will look at the empirical evidence via
  - Event studies.
  - Low-frequency evidence on prices and holdings.

Evidence from key policy announcements

- Event studies are a perfect way to measure the impact of QE on prices if (i) the announcement captures the full surprise, (ii) markets directly incorporate all information into prices.
- The main concern is, however, that policies are to some extent anticipated. Hence, we may see a change in prices, but it is harder to tell what the innovation exactly is (that is, what did the market expect)?
- Three key papers in this area:
  - Gagnon, Raskin, Remache, and Sack (2010).
  - Krishnamurthy and Vissing-Jorgensen (2011).
  - D'Amico, English, Lopez-Salido, Nelson (2012).

- Gagnon, Raskin, Remache, and Sack (2010) focus on QE1 and 7 baseline announcements:

    1. Nov 25-08: The initial LSAP announcement in which the Federal Reserve announced it would purchase up to $100 billion in agency debt, and up to $500 billion in agency MBS.
    2. Dec 1-08: Chairman Bernanke's speech saying the Fed "could purchase longer-term Treasury securities . . . in substantial quantities."
    3. Dec-08/Jan-09: FOMC statements, indicating the consideration to expand purchases of agency securities and start purchases of longer-term Treasuries.
    4. March-09: FOMC statement, including the decision to purchase "up to" $300 billion of longer-term Treasury securities, and to increase the size of agency debt and agency MBS purchases to "up to" $200 billion and $1.25 trillion, respectively.
    5. Aug-09: FOMC statement, which dropped the "up to" language qualifying the maximum amount of Treasury purchases, and announced a gradual slowing in the pace of these purchases;
    6. Sept-09: FOMC statement, which dropped the "up to" language qualifying the maximum amount of agency MBS purchases, and announced a gradual slowing in the pace of agency debt and MBS purchases.
    7. Nov-09: FOMC statement, which stated that the FOMC would purchase "around $175 billion of agency debt."

- Methodology: Look at prices from close of the previous day to the close of the announcement day. These announcements always occur during the trading day.

Table 1: Interest Rate Changes around Baseline and Extended Event Set Announcements

Date	Event	2y UST	10y UST	10y Agy	Agy MBS#	10y TP	10y Swap	Baa Index
11/25/2008*	Initial LSAP Announcement	-2	-22	-58	-44	-17	-29	-18
12/1/2008*	Chairman Speech	-8	-19	-39	-15	-17	-17	-12
12/16/2008*	FOMC Statement	-9	-26	-29	-37	-12	-32	-11
1/28/2009*	FOMC Statement	10	14	14	11	9	14	2
3/18/2009*	FOMC Statement	-22	-47	-52	-31	-40	-39	-29
4/29/2009	FOMC Statement	1	10	-1	6	6	8	-3
6/24/2009	FOMC Statement	10	6	3	2	4	4	5
8/12/2009*	FOMC Statement	-2	5	4	2	3	1	2
9/23/2009*	FOMC Statement	1	-3	-3	-1	-1	-5	-4
11/4/2009*	FOMC Statement	-2	6	8	1	5	5	3
12/16/2009	FOMC Statement	-2	1	0	-1	1	1	-1
1/28/2010	FOMC Statement	-6	-1	0	-1	1	-1	0
1/6/2009	Minutes Release	0	-4	3	-17	-1	-9	-14
2/18/2009	Minutes Release	9	11	4	6	8	9	16
4/8/2009	Minutes Release	2	-4	-7	-9	-4	-6	-6
5/20/2009	Minutes Release	-5	-5	-5	-7	-4	-4	-10
7/15/2009	Minutes Release	7	13	16	16	10	16	7
9/2/2009	Minutes Release	-1	-6	-6	-4	-7	-8	-5
10/14/2009	Minutes Release	1	7	10	3	7	7	8
11/24/2009	Minutes Release	0	-5	-5	-9	-5	-6	-3
1/6/2010	Minutes Release	-2	6	5	4	6	7	-1
Baseline Event Set		***-34***	***-91***	***-156***	***-113***	***-71***	***-101***	***-67***
Baseline Set + All FOMC		***-19***	***-62***	***-140***	***-123***	***-50***	***-83***	***-74***
Cumulative Change: 11/24/08 to 1/28/2010		***-39***	***30***	***-96***	***-109***	***21***	***20***	***-482***

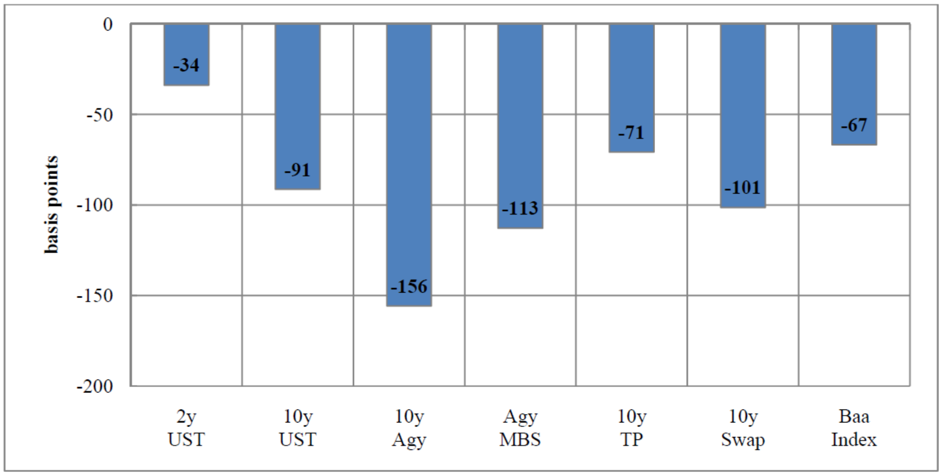
* Included in the baseline event set.

\# Two-day change for agency MBS on March 18, 2009 due to a Bloomberg data error.

- Different yields: UST = Treasuries; Agy = agency debt yield; TP = Term premium measure.

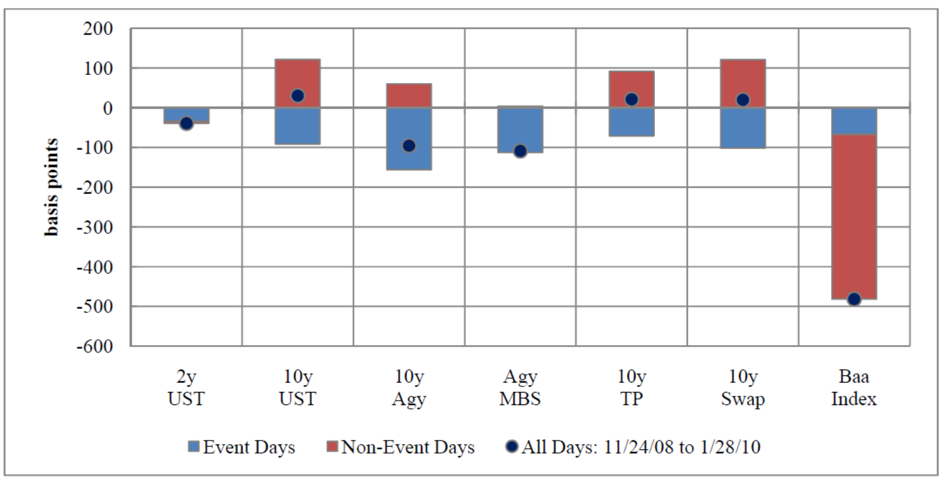
- Yields fall significantly on this set of days. Consistent with duration risk being removed from the market, the term premium falls.

Chart 5: Cumulative Interest Changes on Baseline Event Set Days



Source: Bloomberg, Barclay's Capital

Chart 6: Cumulative Changes since November 2008, Event vs. non-Event Days



Source: Bloomberg, Barclay's Capital

- Note, again, that there is quite some action on non-announcement days as well. The 10-year yield, for instance, drifts up more on non-announcement days than it declines on announcement days. The net effect is positive.

- Krishnamurthy and Vissing-Jorgensen (2011) is the classic paper in this literature and analyzes the channels in great detail by studying the responses of a wide range of asset prices around key event dates.

- **Goal:** Disentangle seven channels. Summary of the channels and their main predictions for asset prices

  1. Duration risk channel.
     The yields of all long-term, nominal assets decline including Treasuries, corporate bonds, and mortgages.
     The effect should also be larger for long-duration assets.

     Motivated by the Vayanos and Vila (2009) model that we discussed earlier.
  2. Liquidity channel.
     Reserves are the most liquid asset. By swapping long-term assets (Treasuries or MBA, which are less liquid) for reserves, the liquidity premium embedded in Treasuries declines, meaning that Treasury yields should *increase*.
     QE should have a larger effect for more liquid assets, which typically embed a liquidity premium, relative to less liquid assets.
  3. Safety premium channel.
     By removing safe assets from markets, the safety premium increases and the yield on safe assets (Treasuries, agency debt, and high-grade corporate bonds) declines. The largest effect for the safest assets, where they argue that Baa (the cutoff between investment- and speculative grade debt) is the relevant cutoff. Based on the results of Krishnamurthy and Vissing-Jorgensen (2012).

4. Signalling channel.
   If QE signals the commitment of the central bank to keep rates low for a long period of time, this affects all fixed income instruments. Expectations (albeit under the risk-neutral measure) of future interest rates can be measured via Federal Funds futures contracts. The signalling channel should have most impact on short- to medium-term rates, as opposed to the very long-term yields as the Central Bank's commitment is until the economy recovers.

5. Prepayment risk channel.
   QE1 involves large purchases of MBS. If MBS markets are segmented, as argued by Gabaix, Krishnamurthy, and Vigneron (2007), then this reduces the risk premium associated with prepayment risk (similar to the Vayanos and Vila, 2009, logic for interest rate risk). QE1 should lower MBS yields relative to other yields. QE2, which does not involve MBS purchases, does not affect MBS yields beyond the interest rate effect.

6. Default risk channel.
   QE may affect the quantity and price of default risk if QE succeeds in stimulating the economy. We should see this in the price of CDS contracts.

7. Inflation channel.
   If QE is expansionary, it increases inflation expectations. QE increases the rate on inflation swaps and the inflation expectations implied by the difference between nominal yields and TIPS.

- Putting it all together,

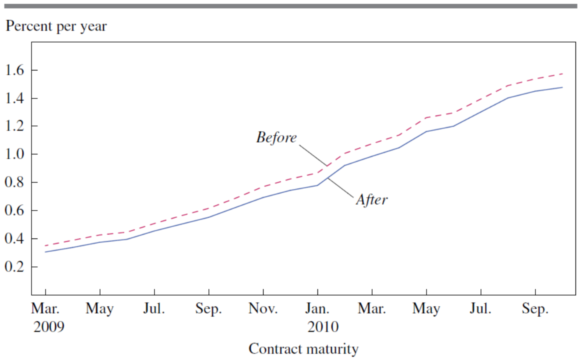
$$
\begin{aligned}
r_{risky,\,illiq\,,long-term} = E\left[i_{safe,\,liq\,,short-term}\right] - \pi^{e} \\
+ Duration \times P_{DurationRisk} \\
+ Illiquidity \times P_{Liquidity} \\
+ LackofSafety \times P_{Safety} \\
+ DefaultRisk \times P_{DefaultRisk} \\
+ PrepaymentRisk \times P_{PrepaymentRisk}.
\end{aligned}
$$

- Selecting event dates:

  "*Gagnon and others (2010) identify eight event dates beginning with the November 25, 2008, announcement of the Federal Reserve's intent to purchase \$500 billion of agency MBSs and \$100 billion of agency debt and continuing into the fall of 2009. We focus on the first five of these event dates (November 25, December 1, and December 16, 2008, and January 28 and March 18, 2009), leaving out three later event dates on which only small yield changes occurred.*"

- Measuring the signalling channel via Federal funds futures:

Figure 3. Yield Curves Calculated from Federal Funds Futures before and after QE1 Event Days



Source: Bloomberg data.
a. Yields are computed the day before each QE1 event date and again the day after. All the before-event yields are then averaged across events, and likewise for the after-event yields.

Table 2. Changes in Federal Funds Futures Yields around QE1 and QE2 Event Dates[a]
Basis points

	Federal funds futures, contract maturity			
Date[b]	*3rd month*	*6th month*	*12th month*	*24th month*
QE1[c]	−6	−5	−8	−16
Nov. 25, 2008	−6	−3	−7	−20
Dec. 1, 2008	−13	−15	−10	−11
Dec. 16, 2008	−1	−1	−1	19
Jan. 28, 2009	−2	−4	−8	−11
Mar. 18, 2009	−28*	−27	−33**	−40
Sum[d]				

- They attribute about 40bp to the signalling channel.

Table 1. Changes in Treasury, Agency, and Agency MBS Yields around QE1 Event Dates[a]
Basis points

		Treasury yields (constant maturity)					*Agency (Fannie Mae) yields*				*Agency MBS yields*[b]	
Date	*Event*	*30-year*	*10-year*	*5-year*	*3-year*	*1-year*	*30-year*	*10-year*	*5-year*	*3-year*	*30-year*	*15-year*
Nov. 25, 2008	Initial announcement	−24	−36	−23	−15	−2	−57	−76	−57	−42	−72	−88
Dec. 1, 2008	Bernanke speech	−27	−25	−28	−15	−13	−52	−67	−50	−33	−14	12
Dec. 16, 2008	FOMC statement	−32	−33	−15	−4	−5	−37	−39	−26	−25	−26	−16
Jan. 28, 2009	FOMC statement	31	28	28	19	4	33	28	27	14	31	20
Mar. 18, 2009	FOMC statement	−21	−41	−36	−24	−9	−31	−45	−44	−35	−27	−16
Sum of above five dates[c]		−73*	−107**	−74	−39	−25**	−144**	−200***	−150***	−123***	−107*	−88

Sources: FRED, Federal Reserve Bank of St. Louis; Bloomberg.
a. All changes are over 2 days, from the day before to the day after the event. Asterisks denote statistical significance at the ***1 percent, **5 percent, and *10 percent level.
b. Averages across current-coupon Ginnie Mae, Fannie Mae, and Freddie Mac MBSs.
c. May differ from the sum of the values reported for individual dates because of rounding.

- Consistent with the duration channel, longer-term yields fall more. However, yields across types of bonds (Treasuries, agencies), the responses are quite different so this is not the full story.

- If we compare Treasuries and agency yields, then they have the same credit risk, but agencies are less liquid. Agency yields fall a lot more, consistent with a lower liquidity premium embedded in Treasuries.

- Agencies are primarily exposed to the duration (limited explanatory power), signalling, and safety channel. The large response seems most consistent that QE has an important effect on the safety premium.

- The decline in MBS yields may be consistent with a reduction in prepayment risk. Note: Because of prepayment risk, the duration of a 30-year MBS is actually around 7 years.

Table 3. Changes in Corporate Yields, Unadjusted and Adjusted by Credit Default Swap Rates, around QE1 Event Dates[a]
Basis points

	Corporate yields											
	Long-term						*Intermediate-term*					
Date[b]	*Aaa*	*Aa*	*A*	*Baa*	*Ba*	*B*	*Aaa*	*Aa*	*A*	*Baa*	*Ba*	*B*
Nov. 25, 2008	−28	−18	−23	−19	−4	4	−17	−15	−18	−18	1	−47
Dec. 1, 2008	−24	−24	−21	−17	−13	28	−21	−15	−18	−8	−5	6
Dec. 16, 2008	−43	−37	−45	−39	1	−11	−19	−21	−24	−27	−28	−42
Jan. 28, 2009	34	17	17	14	−16	−25	12	8	7	3	−32	−25
Mar. 18, 2009	−16	−21	−21	−20	−28	−39	−43	−50	−39	−26	−18	−22
Sum[c]	−77	−83**	−93**	−81**	−60**	−43	−88**	−93**	−92**	−76**	−82***	−130***
	Credit default swap rates[d]											
	10-year maturity						*5-year maturity*					
Nov. 25, 2008	−1	10	−17	−13	−31	−798	−1	−6	−20	−18	−32	−573
Dec. 1, 2008	1	0	9	11	21	1	1	3	13	7	28	33
Dec. 16, 2008	−2	−8	−18	−17	−23	−308	−2	−15	−20	−21	−40	−172
Jan. 28, 2009	−3	−15	−6	−13	−26	−231	−3	−7	−9	−11	−27	−255
Mar. 18, 2009	−2	−1	0	−7	−18	−18	−2	8	2	−8	−27	−25
Sum[c]	−7***	−14	−32	−40*	−78*	−1,354**	−6***	−17	−33	−51**	−98*	−991**

(continued)

	Adjusted corporate yields[e]											
	Long-term						*Intermediate-term*					
Date[b]	*Aaa*	*Aa*	*A*	*Baa*	*Ba*	*B*	*Aaa*	*Aa*	*A*	*Baa*	*Ba*	*B*
Nov. 25, 2008	−27	−28	−6	−6	27	802	−16	−9	2	0	33	526
Dec. 1, 2008	−25	−24	−30	−28	−34	27	−22	−18	−31	−15	−33	−27
Dec. 16, 2008	−41	−29	−27	−22	24	297	−17	−6	−4	−6	12	130
Jan. 28, 2009	37	32	23	27	10	206	15	15	16	14	−5	230
Mar. 18, 2009	−14	−20	−21	−13	−10	−21	−41	−58	−41	−18	9	3
Sum[c]	−70	−69	−61	−41	18	1,311**	−82*	−76	−59	−25	16	861**

Sources: Authors' calculations using data from Barclays, Credit Market Analysis (CMA), the Mergent Fixed Investment Securities Database (FISD), and the Trade Reporting and Compliance Engine (TRACE) of the Financial Industry Regulatory Authority.
a. All changes are over 2 days, from the day before to the day after the event. Asterisks denote statistical significance at the ***1 percent, **5 percent, and *10 percent level.
b. See table 1 for descriptions of the events on these dates.
c. May differ from the sum of the values reported for individual dates because of rounding.
d. Constructed using CMA data and ratings from FISD; changes are value-weighted averages using information on issue sizes from FISD and prices from TRACE.
e. Change in the unadjusted corporate yield minus the change in the corresponding CDS rate.

- CDS spreads fall significantly for lower-rated bonds, consistent with a reduction in the pricing and quantity of default risk.

- Agencies and Treasuries are safe assets. Agency yields fall by a lot. In addition, highly-rated bond yields, adjusted for CDS (third panel above), also decline a lot, again consistent with a safety premium channel.

Table 4. Changes in Inflation Swap Rates, TIPS Yields, and Implied Interest Rate Volatility around QE1 Event Dates[a]
Basis points

	Inflation swap rates				*TIPS real yield (constant maturity)*			*Implied interest rate volatility*[c]
Date[b]	*30-year*	*10-year*	*5-year*	*1-year*	*20-year*	*10-year*	*5-year*	
Nov. 25, 2008	1	–6	–28	48	–22	–43	5	1
Dec. 1, 2008	15	27	12	–40	–38	–34	–52[d]	–7
Dec. 16, 2008	4	37	35	–17	–45	–57	–83	–20
Jan. 28, 2009	14	15	–6	5	15	6	13	0
Mar. 18, 2009	2	22	24	45	–45	–59	–43	–11
Sum[e]	35**	96**	38	41	–135***	–187***	–160**	–38***

Sources: FRED, Federal Reserve Bank of St. Louis; Bloomberg.

a. All changes are over 2 days, from the day before to the day after the event. Asterisks denote statistical significance at the ***1 percent, **5 percent, and *10 percent level.

b. See table 1 for descriptions of the events on these dates.

c. Volatility implied from swaptions as measured using the Barclays implied volatility index.

d. The constant-maturity TIPS data from FRED indicate that the 5-year TIPS fell by 244 bp around this event. We think this is a data error. Using data from FRED on the 5-year and 10-year underlying TIPS with remaining maturities near 5 years around QE1 (the 5-year TIPS maturing April 15, 2013, and the 10-year TIPS maturing January 15, 2014), we found yield changes of –58 bp and –46 bp, respectively. The value reported in the table is the average of these changes.

e. May differ from the sum of the values reported for individual dates because of rounding.

- Evidence from inflation swaps suggests that inflation expectations increased significantly as well, consistent with the inflation channel.

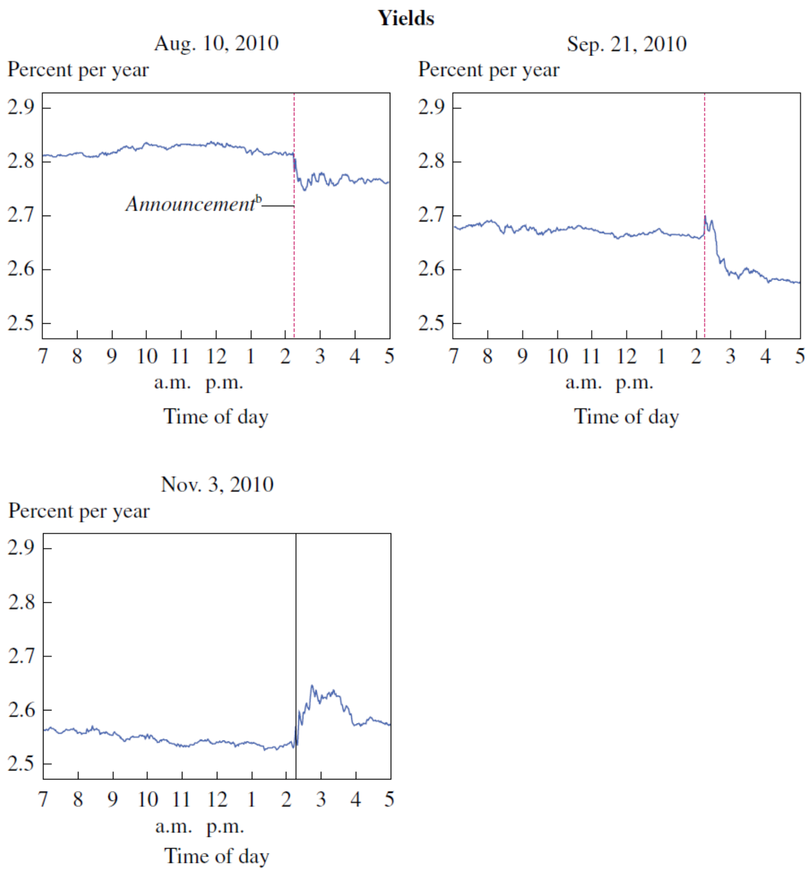
- **Summary:** During QE1, many effects are operating at the same time and it is hard to (quantitatively) disentangle them without precise models or measures of risk exposures. Important channels:

  - Signalling.
  - Increase in the safety premium.
  - Reduction in default and prepayment risk premia.
  - Large effect on inflation expectations.

- Smaller effect for the duration channel.

- For QE2, there are three dates, but yields rise for one:

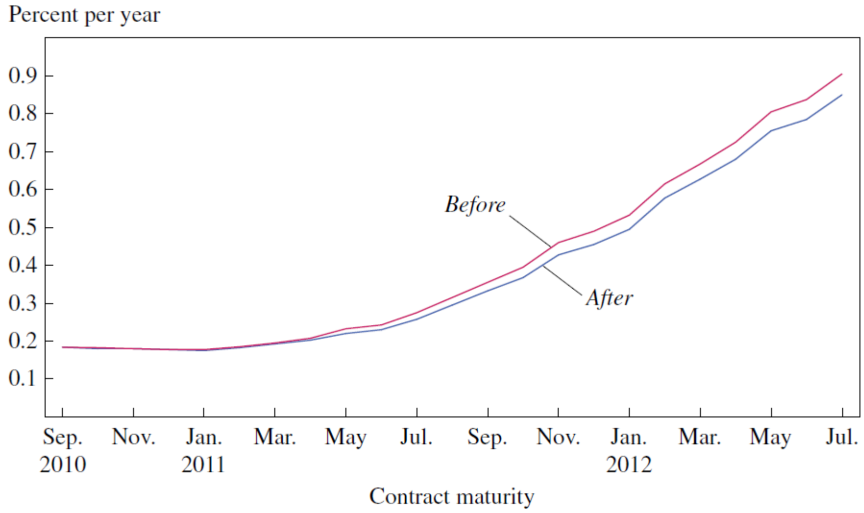
Figure 4. Intraday Yields and Trading Volumes on QE2 Event Days[a]



- They proceed without November 3, 2010:

> *"We do not add in the change from the 11/3 announcement as it is unclear whether only the increase in the yields after the announcement or also also the subsequent decrease was due to QE2."*

Figure 5. Yield Curves Calculated from Federal Funds Futures before and after QE2 Event Days[a]



Source: Bloomberg data.

a. Yields are computed the day before each QE2 event date and again the day after. All the before-event yields are then averaged across events, and likewise for the after-event yields.

- The impact of QE2 is generally smaller. There is evidence of the signalling channel again based on Federal funds futures.

Table 5. Changes in Treasury, Agency, and Agency MBS Yields around QE2 Event Dates[a]
Basis points

	Treasury yields (constant maturity)					*Agency (Fannie Mae) yields*				*Agency MBS yields*[b]	
Date	*30-year*	*10-year*	*5-year*	*3-year*	*1-year*	*30-year*	*10-year*	*5-year*	*3-year*	*30-year*	*15-year*
Aug. 10, 2010											
One-day change	−1	−7	−8	−3	−1	−2	−7	−8	−4	−1	−4
Two-day change	−8	−14	−10	−3	−1	−8	−13	−9	−7	−4	−8
Sep. 21, 2010											
One-day change	−8	−11	−9	−5	0	−8	−11	−9	−6	−8	−8
Two-day change	−13	−16	−10	−5	−1	−14	−16	−10	−6	−4	−5
Nov. 3, 2010											
One-day change	16	4	−4	−2	0	13	5	−5	−3	−4	−4
Two-day change	11	−10	−11	−6	−1	4	−10	−14	−8	−10	−9
Sum of Aug. 10 and Sep. 21[c]											
One-day change	−9*	−18***	−17***	−8***	−1	−9**	−17***	−17***	−10***	−9*	−12***
Two-day change	−21***	−30***	−20***	−8***	−2	−22***	−29***	−20***	−13***	−8	−13**

Sources: FRED, Federal Reserve Bank of St. Louis; Bloomberg.
a. Dates are those of FOMC statements regarding QE2. Asterisks denote statistical significance at the ***1 percent, **5 percent, and *10 percent level.
b. Averages across current-coupon Ginnie Mae, Fannie Mae, and Freddie Mac MBSs.
c. May differ from the sum of the values reported for individual dates because of rounding.

- Changes in MBS yields similar to the signalling channel, so no evidence of a reduction in prepayment risk.
- As part of QE2, the FED only purchased Treasuries, but no MBS.
- This suggests that demand may be relative inelastic and rebalancing across asset classes is limited.
- More evidence in Di Maggio, Kermani, and Palmer (2016) suggesting that the effects are quite local in the markets in which the FED purchased securities.
- They provide additional evidence in support of the safety channel and an increase in inflation expectation, but not much beyond that.
- Compared to QE1, it is much harder to identify the effects of QE2 as the changes in yields are generally smaller.

- As further evidence of local effects, see D'Amico, English, Lopez-Salido, Nelson (2012).

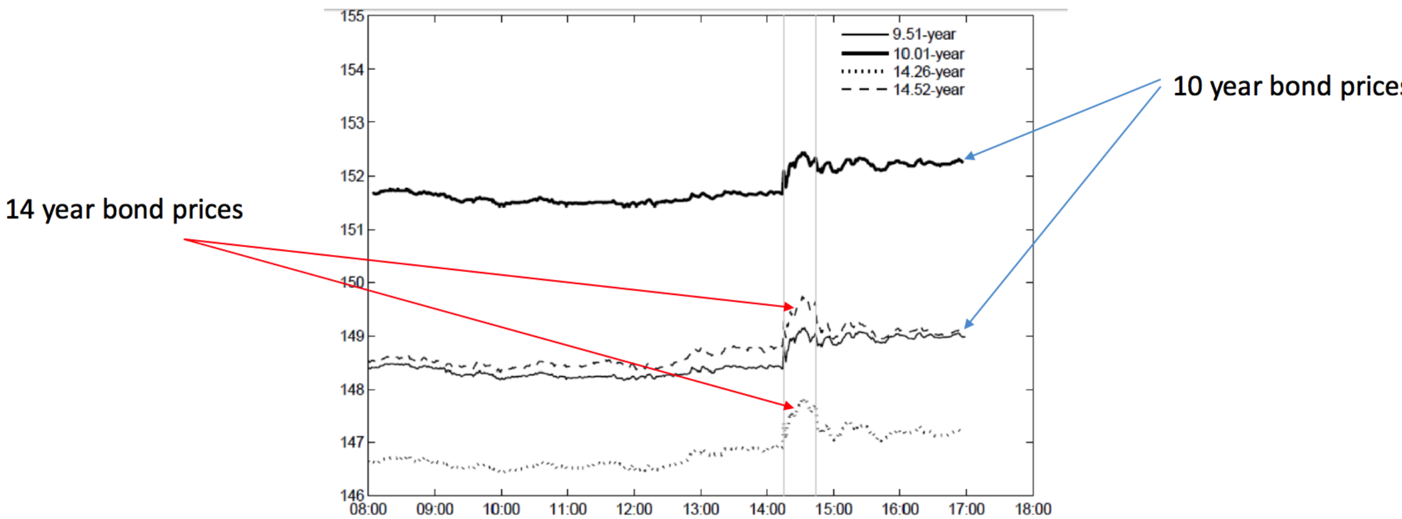


Figure 3: CUSIP-level intraday prices on August 10, 2010:
Vertical lines indicate time of announcements.

August 10, 2010 Fed announcement regarding Treasury purchases

- FOMC meeting of August 10, 2010. In its statement after that meeting, the FOMC announced (at 2.15 p.m.) that principal payments from agency securities would be reinvested in longer-term Treasury securities.

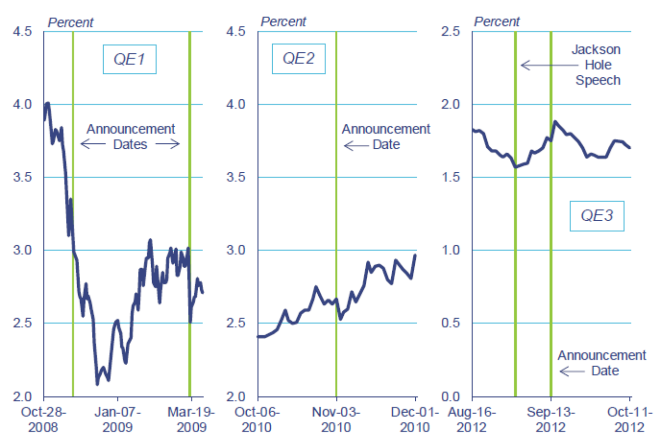
- At 2.45 p.m., the Federal Reserve Bank of New York (FRBNY) issued a statement indicating that the purchases underlying the reinvestment policy would be concentrated in the two- to ten-year sector of the nominal Treasury yield curve.

- Changes over this half-hour interval in market expectations highlight the local demand effects.

- Although yields do move substantially on these days, it is also important to recognize they move a great deal on other days.

- A challenge with the event-study methodology is that we need to identify the right dates, while changes in purchase programs are oftentimes widely discussed in advance.

Figure 4: Quantitative Easing Announcements and Ten-Year Treasury Yields



Source: Federal Reserve Board, Haver Analytics

Evidence based on prices and quantities

- Koijen, Koulischer, Nguyen, and Yogo (2017) and Koijen, Koulischer, Nguyen, and Yogo (2021) study the impact of QE in terms of both quantities and prices using new security-level data on asset holdings of investor sectors across euro-area countries.