Demand System Asset Pricing
A micro-founded asset demand system

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Towards an empirically-tractable model of demand

- Wish list for our model:
  1. Nests modern portfolio theory as a special case.
  2. Empirically tractable.
  3. Sufficiently flexible to allow for inelastic demand curves.

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$$w = \frac{1}{\gamma}\Sigma^{-1}\mu.$$

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- If we model $\mu(n)$ as a function of characteristics of stock n, $x(n)$, as in modern empirical asset pricing, it seems intractable as characteristics of all stocks matter (via Σ^{-1}).
- Key insight: Solution simplifies under realistic assumptions to

$$w(n) = \frac{b'x(n)}{c},$$

where c encodes the information of all other stocks.

Various micro-foundations lead to a demand system

- Various micro-foundations.
  - Mean-variance portfolio choice (Markowitz 1952).
  - Portfolio choice with hedging demand (Merton 1973).
  - Private information and imperfect competition (Kyle 1989).
  - Heterogeneous beliefs.
  - Institutional asset pricing with constraints.
  - Direct preferences for characteristics such as ESG.
- Can be expressed as the same portfolio demand function (see KRY23).
- However, demand elasticities depend on structural parameters in different ways.

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- We have $i = 1, \ldots, I_x$, $x = Q, F$, investors of each type.
- Investors have CARA preferences

$$\max_{\mathbf{q}_i} \mathbb{E}\left[-\exp\left(-\gamma_i A_{1i}\right)\right],$$

with risk aversion coefficients $\gamma_i = \frac{1}{\tau_i A_{i0}}$ and initial assets A_{i0}.

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  with risk aversion coefficients $\gamma_i = \frac{1}{\tau_i A_{i0}}$ and initial assets A_{i0}.
- Investors allocate capital to $n = 1, \ldots, N$ assets.
- Intra-period budget constraint:

$$A_{0i} = \boldsymbol{q}_i' \boldsymbol{P}_0 + Q_i^0,$$

- Dividends are given by $\boldsymbol{D}_1$, which equal $\boldsymbol{P}_1$ in a static model.

Beliefs: Quant investors (KY19)

- Let $\boldsymbol{R}_1 = \boldsymbol{P}_1 - \boldsymbol{P}_0$ be the (dollar) return.
- Quants reason in terms of factor models and try to discover alpha as a function of asset characteristics

$$\begin{aligned} \boldsymbol{R}_1 &= \boldsymbol{a}_i + \boldsymbol{\beta}_i R_1^m + \boldsymbol{\eta}_1, \\ \boldsymbol{\mu}_i &= \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i \Lambda, \end{aligned}$$

  where $\boldsymbol{\mu}_i = \mathbb{E}_i\left[\boldsymbol{R}_1\right]$ and $\mathrm{Var}\left(\boldsymbol{\eta}_1\right) = \sigma^2 \boldsymbol{I}$.
- Hence, the covariance matrix of returns is

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- Key: Alphas and betas are affine in characteristics,

$$\begin{aligned} \beta_i(n) &= \boldsymbol{\lambda}_i^{\beta\prime} \boldsymbol{x}(n) + \nu_i^{\beta}(n), \\ \alpha_i(n) &= \boldsymbol{\lambda}_i^{\alpha\prime} \boldsymbol{x}(n) + \nu_i^{\alpha}(n). \end{aligned}$$

Beliefs: Fundamental investors (KRY23)

- Let $\boldsymbol{R}_1^F = \boldsymbol{D}_1 - \boldsymbol{P}_0$ be the long-run fundamental return.
- Fundamental investors think about the long-run expected growth rate of fundamentals and their riskiness

$$\boldsymbol{D}_1 = \boldsymbol{g}_i + \boldsymbol{\rho}_i F_1 + \boldsymbol{\epsilon}_1,$$

  where $\mathrm{Var}\left(\boldsymbol{\epsilon}_1\right) = \sigma^2 \boldsymbol{I}$.
- Hence, the covariance matrix of long-horizon returns is

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- Key: Factor loadings and expected growth are affine in characteristics,

$$\begin{aligned} \rho_i(n) &= \boldsymbol{\lambda}_i^{\rho\prime} \boldsymbol{x}(n) + \nu_i^{\rho}(n), \\ g_i(n) &= \boldsymbol{\lambda}_i^{g\prime} \boldsymbol{x}(n) + \nu_i^{g}(n). \end{aligned}$$

Demand curves

- The quant's optimal portfolio is

$$\boldsymbol{q}_i^Q = \frac{1}{\gamma_i}\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\mu}_i.$$

- The optimal portfolio of the fundamental investor is

$$\boldsymbol{q}_i^F = \frac{1}{\gamma_i}\left(\boldsymbol{\Sigma}_i^F\right)^{-1}(\boldsymbol{g}_i - \boldsymbol{P}_0).$$

Key insight

- In both cases, the demand curve takes the form

$$\boldsymbol{q}_i = \frac{1}{\gamma}\left(\boldsymbol{v}_i\boldsymbol{v}_i' + \sigma^2 \boldsymbol{I}\right)^{-1}\boldsymbol{m}_i.$$

Key insight

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$$\boldsymbol{q}_i = \frac{1}{\gamma}\left(\boldsymbol{v}_i \boldsymbol{v}_i' + \sigma^2 \boldsymbol{I}\right)^{-1} \boldsymbol{m}_i.$$

- Using the Woodbury matrix identity, we have

$$\begin{aligned} \boldsymbol{q}_i &= \frac{1}{\gamma\sigma^2}\left(\boldsymbol{I} - \frac{\boldsymbol{v}_i \boldsymbol{v}_i'}{\boldsymbol{v}_i' \boldsymbol{v}_i + \sigma^2}\right) \boldsymbol{m}_i \\ &= \frac{1}{\gamma\sigma^2}\left(\boldsymbol{m}_i - c_i \boldsymbol{v}_i\right), \end{aligned}$$

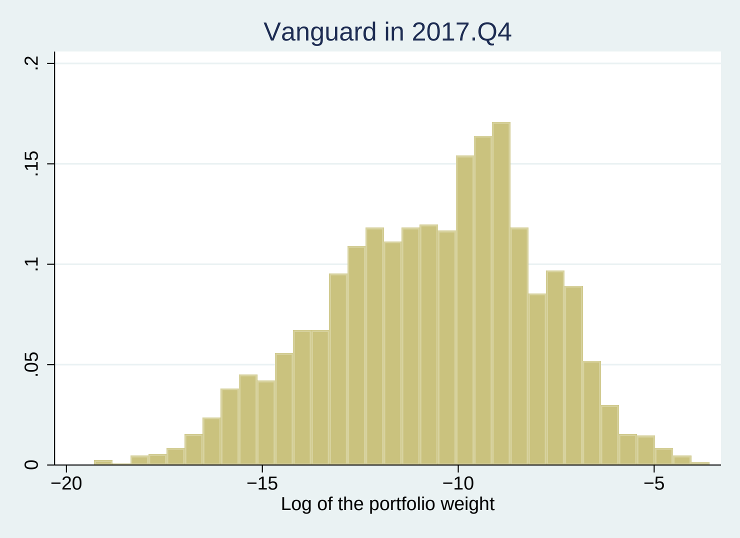
  where $c_i = \frac{\boldsymbol{v}_i' \boldsymbol{m}_i}{\boldsymbol{v}_i' \boldsymbol{v}_i + \sigma^2}$ is a scalar that encodes the information of all other stocks.
- The demand for stock n only depends on the characteristics of stock n and a common scalar, c_i.
- Intuition: The factor exposure and alpha are sufficient statistics for the attractiveness of stock n.

Three implementations of the mean-variance portfolio

- Estimate mean-variance portfolio among stocks in the S&P 500 index, subject to short-sale constraints.
  1. Benchmark: Unrestricted mean and covariance matrix.
  2. Factor structure: Impose FF 5-factor model on mean and covariance.
  3. Characteristics: Exponential-linear function of characteristics.

Statistic	Benchmark	Factor structure	Characteristics
Mean (%)	1.1	1.5	1.5
Standard deviation (%)	4.3	6.2	5.9
Certainty equivalent (%)	1.0	1.3	1.3
Correlation:			
Factor structure	0.54		
Characteristics	0.50	0.93	

Empirical regularity: Holdings are log-normally distributed



An empirically tractable asset demand system

- Investors select stocks in a choice set $\mathcal{N}_i \subset \{1, \ldots, N\}$.
- The portfolio weight on stock n is

$$w_i(n) = \frac{\delta_i(n)}{1 + \sum_{m \in \mathcal{N}_i} \delta_i(m)},$$

where

$$\delta_i(n) = \exp(b_{0,i} + \beta_{0,i} me(n) + \beta_{1,i}' x(n)) \epsilon_i(n).$$

and

  - $b_{0,i}$: Controls the fraction invested in the outside asset.
  - $\beta_{0,i} < 1$: Controls the price elasticity of demand.
  - $me(n)$: Log market equity.
  - $x(n)$: Stock characteristics (e.g., log book equity, profitability).
  - $\beta_{1,i}$: Demand for characteristics.
  - $\epsilon_i(n) \geq 0$: Latent demand.

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- A passive portfolio using market weights is replicated by
  - $\beta_{0,i} = 1$
  - $\beta_{1,i} = 0$
  - $\epsilon_i(n) = 1$.

Solve for asset prices by imposing market clearing

- Market clearing

$$ME(n) = \sum_{i=1}^{I} A_i w_i(n, \mathbf{me}, \mathbf{x}, \epsilon).$$

- KY19 show that a unique equilibrium exists if demand is downward sloping for all investors (i.e., $\beta_{0,i} < 1$).
- Despite this high-dimensional, nonlinear system in asset prices, we will discuss a simple algorithm to solve it quickly.

Lessons learned

- Assumptions commonly made in empirical asset pricing,
  1. Factor loadings depend on characteristics,
  2. Alphas depend on characteristics,

  have a convenient implication for optimal portfolios.
- Optimal demand for stock n only depends on that stock's characteristics and a scalar that encodes the information of all other stocks.
- We introduced an empirically-tractable model of the demand curve that adopts this structure and matches the lognormal property of portfolio weights.